A BODY-ORIENTED METHOD FOR DYNAMIC MODELING AND ADAPTIVE CONTROL OF FULLY PARALLEL ROBOTS

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Abstract: In this paper we propose a method based on the virtual work principle to find a linear form of the dynamic equation of fully parallel robots. Compared to other methods, it has the advantage that it does not need to open the closed loop structure into a tree-structure robot. It considers rather each body separately using its Jacobian matrix to project the forces into the joint space of the robot. Thus, simplifications can be made at the very beginning of the modeling, that is very usefull for real-time nonlinear adaptive control implementation. As an example, the method is applied to the Hexaglide, a parallel machine tool with 6 degree-of-freedom. Based on this model, a simulation of a non-linear adaptive controller is performed, demonstrating the possibility to apply nonlinear adaptive control to complex parallel robots.

Keywords: robot control, robot dynamics, adaptive control, parameter identification, parallel robot

1. INTRODUCTION

The dynamic equation of robots is linear in its dynamic parameters (Khalil and Kleinfinger, 1987; Khosla, 1986). This property has been exploited to identify the dynamic parameters of robots using Least-Square techniques (An *et al.*, 1985; Khosla and Kanade, 1985) or nonlinear adaptive control (Craig, 1988; Slotine and Li, 1987). Many solutions have been proposed to find a linear form of the dynamic equation of serial (Gautier and Khalil, 1990; Khosla, 1986) and tree-structure robots (Khalil *et al.*, 1989). For manipulators with closed loop chains, the dynamic equation is generally difficult to establish. It is thus not easy to write it in a linear form. Solutions have been proposed (Walker, 1990; Goldenberg *et al.*, 1992; Khalil and Bennis, 1995) in which the model is usually obtained by opening the loops in order to form a tree-structure robot. The dynamics equation is then derived using classical techniques developed for serial robots (Khalil *et al.*, 1989) and a loop closure constraint equation to eliminate the non-actuated joints from the tree-structure dynamic model. The method has been applied to planar parallel mechanisms, but becomes very cumbersome for spacial parallel robots. The problem has been recently addressed by (Bhattacharya *et al.*, 1997) who proposed a solution based on an energy model of the parallel structure. It results in a parallelism in the dynamic equations which can be exploited efficiently in a multiprocessor controller structure. It remains however very computationally intensive.

In this paper we propose a new method for finding the dynamic equation in an explicit linear function of the the dynamic parameters. It is based on the virtual work principle and uses Jacobian matrices to "project" the forces of each body in the space of the active joints. It has the following advantages:

- The robot does not need to be opened in an equivalent tree-structure mechanism.
- Simplifying hypotheses can be used at the very beginning of the modeling already.
- It leads to simple equations that can be used for real time implementation of nonlinear adaptive control.

In this paper, we first present the proposed method to get a linear form of the dynamics equation. The method is applied to the Hexaglide, a parallel platform with 6 dof and then used in a nonlinear adaptive control scheme. Finally, simulations are performed showing the efficiency of nonlinear adaptive control of complex parallel robots.

2. DYNAMIC EQUATION

2.1 *Definitions*

A robot is a system of N rigid bodies connected together in order to form a kinematic chain. Let q be the vector of active joint variables and $\mathcal{X}$ the vector of operational space variables describing the pose of the end-effector in a reference frame $\mathcal{O}$. A frame $\mathcal{C}_i$ is attached to the centre of mass of body i. Let m_i and I_i be the mass and inertia tensor of body i, and x_i the position of its centre of mass in the reference frame $\mathcal{O}$. $\dot{x}_i$ and ω_i are its linear and angular velocity respectively. The rotation matrix between body frame $\mathcal{C}_i$ and the inertial reference frame $\mathcal{O}$ is noted ${}^{O}_{i}R$. For each body, the linear and angular velocity of the center of mass can be expressed as a function of the joint velocity vector $\dot{q}$ by the mean of the *basic Jacobian* (?) defined by:

$$\dot{\mathcal{X}}_i = \begin{bmatrix} \dot{x}_i \\ \omega_i \end{bmatrix} \equiv J_{0i}\,\dot{q} \qquad (1)$$

This basic Jacobian can also be used to transform a virtual displacement of the joint space vector into a virtual displacement of body i.

2.2 *Virtual work principle*

Using the virtual work (or d'Alembert) principle, the dynamics of a system of N rigid bodies can be described by (Codourey and Burdet, 1997):

$$\sum_{i=1}^{N} J_{0i}^T \left(\mathcal{F}_i^{inertia} - \mathcal{F}_i^{applied}\right) = 0 \qquad (2)$$

with

$$\mathcal{F}_i^{inertia} \equiv \begin{bmatrix} m_i \ddot{x}_i \\ I_i \dot{\omega}_i + \omega_i \times I_i \omega_i \end{bmatrix}, \qquad (3)$$

$$\mathcal{F}_i^{applied} \equiv \begin{bmatrix} F_i \\ T_i \end{bmatrix}, \qquad (4)$$

and where J_{0i} is the basic Jacobian as defined in equation (1) and F_i, T_i the applied forces and torques (e.g. motor torques, gravity, friction).

2.3 *Extracting the actuator forces*

In order to write equation (2) in a linear form of the dynamic parameters, the actuator forces τ must firstly be extracted. The wrench of applied forces and torques $\mathcal{F}_i^{applied}$ can be splitted in an actuator wrench $\mathcal{F}_i^{a}$ and an external wrench $\mathcal{F}_i^{ext}$. Equation (2) can thus be rewritten as follows:

$$\sum_{i=1}^{N} J_{0i}^T\, \mathcal{F}_i^{a} = \sum_{i=1}^{N} J_{0i}^T \left(\mathcal{F}_i^{inertia} - \mathcal{F}_i^{ext}\right), \qquad (5)$$

or also,

$$J_\tau \tau = \sum_{i=1}^{N} J_{0i}^T \left(\mathcal{F}_i^{inertia} - \mathcal{F}_i^{ext}\right) \qquad (6)$$

where τ is the actuator torque vector and J_τ is defined as follows (Codourey and Burdet, 1997):

$$J_\tau \equiv \left[\, J_{A1}^T \Upsilon_1 \;\; J_{A2}^T \Upsilon_2 \;\; \ldots \;\; J_{Am}^T \Upsilon_m \,\right], \qquad (7)$$

with m the number of actuators, J_{Ai} the Jacobian matrix linking the velocity of frame $\mathcal{A}_i$ attached to the motor and the joint velocity vector $\dot{q}$, and

$$\Upsilon_i \equiv \begin{bmatrix} 0 \\ 0 \\ \epsilon_i \\ 0 \\ 0 \\ \bar{\epsilon}_i \end{bmatrix}, \qquad (8)$$

$$\epsilon_i \equiv \begin{cases} 0 \text{ for revolute joint} \\ 1 \text{ for prismatic joint} \end{cases}, \quad \bar{\epsilon}_i = 1 - \epsilon_i\,.$$

Using this definition, the dynamic equation (2) can finally be written as:

$$\tau = J_\tau^{-1} \left[\sum_{i=1}^{N} J_{0i}^T \left(\mathcal{F}_i^{inertia} - \mathcal{F}_i^{ext}\right)\right]. \qquad (9)$$

3. LINEAR FORM OF THE DYNAMIC EQUATION

Assuming that the kinematics of the system of N rigid bodies is known, its dynamic equation can be written in a linear form of the dynamic parameters (Khosla, 1986; Sciavicco and Siciliano, 1996):

$$\tau = \Psi(q, \dot{q}, \ddot{q})\, p\,, \tag{10}$$

where Ψ is the *dynamic matrix* and p the *dynamic parameter vector* containing functions of the lengths, masses and inertias.

3.1 *Inertial forces on a rigid body*

Inertial forces are usually described relatively to the center of mass of a body. If the body is described relatively to an arbitrary frame $\mathcal{A}$ attached to body i, a transformation has to be realized[1] .

We assume that the Jacobian matrix J_A linking the joint velocity $\dot{q}$ to the linear and angular velocities of frame $\mathcal{A}$ is known, as well as its velocities $\dot{x}_A$, ω_A and accelerations.

According to equation (3), inertial forces ${}^C\mathcal{F}_C$ acting at the centre of mass of the body are calculated using the following equation:

$${}^C\mathcal{F}_C = \begin{bmatrix} m\ {}^C\ddot{x}_C \\ {}^CI_C\ {}^C\dot{\omega}_C\ +\ {}^C\omega_C\ \times\ {}^CI_C\ {}^C\omega_C \end{bmatrix} \tag{11}$$

They can be rewritten in a linear form of the dynamic parameters of body i with respect to an arbitrary reference frame $\mathcal{A}$ attached to the body (Codourey and Burdet, 1997):

$${}^A\mathcal{F}_A = \begin{bmatrix} {}^A\ddot{x}_A & \Lambda & 0 \\ 0 & -{}^A\hat{\ddot{x}}_A & \psi_\omega \end{bmatrix} \begin{bmatrix} m \\ m\,r \\ I_p \end{bmatrix}\,, \tag{12}$$

where

$$\Lambda = {}^A\hat{\omega}_A\, {}^A\hat{\omega}_A + {}^A\hat{\dot{\omega}}_A\,, \tag{13}$$

$$\psi_\omega = {}^A\hat{\omega}_A\, {}^A\tilde{\omega}_A + {}^A\tilde{\dot{\omega}}_A\,, \tag{14}$$

with

$$\tilde{\omega} = \begin{bmatrix} \omega_x & \omega_y & \omega_z & 0 & 0 & 0 \\ 0 & \omega_x & 0 & \omega_y & \omega_z & 0 \\ 0 & 0 & \omega_x & 0 & \omega_y & \omega_z \end{bmatrix} \tag{15}$$

and where a hat ($\hat{a}$) over a vector indicates its skew symmetric matrix notation such that $\hat{a}b = a \times b$ for any vector $a = [a_x,\, a_y,\, a_z]^T$, and is defined as:

$$\hat{r} = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix}. \tag{16}$$

m is the mass of the body, $m\,r$ its first order moment vector and $I_p \equiv [I_{xx}, I_{xy}, I_{xz}, I_{yy}, I_{zy}, I_{zz}]^T$ a vector composed of the second order moments (inertia) of the body. Expression (12) is similar to the one obtained by (Goldenberg *et al.*, 1992).

The wrench ${}^A\mathcal{F}_A$ can finally be calculated in reference frame $\mathcal{O}$ as follows:

$${}^O\mathcal{F}_A = \begin{bmatrix} {}^O_AR & 0 \\ 0 & {}^O_AR \end{bmatrix}\ {}^A\mathcal{F}_A \tag{17}$$

3.2 *Linear form*

Assuming that the external forces are composed of gravitational forces only, equation (9) can be rewritten with respect to body points A_i, $i = 1..., N$, as:

$$\tau = J_\tau^{-1} \left[\sum_{i=1}^{N} J_{Ai}^T\ {}^O\mathcal{F}_A - \sum_{i=1}^{N} J_{Oi}^T \begin{bmatrix} m_i G \\ 0 \end{bmatrix} \right], \tag{18}$$

where G is the vector of gravitational acceleration. By adding the acceleration of gravity to the acceleration of body i, this equation can be rearranged as follows:

$$\tau = J_\tau^{-1} \left[\ J_{A_1}^T\Omega_1\ \ J_{A_2}^T\Omega_2\ \dots\ J_{A_N}^T\Omega_N\ \right] \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_N \end{bmatrix},$$

where

$$\Omega_i \equiv \begin{bmatrix} {}^O_{A_i}R & 0 \\ 0 & {}^O_{A_i}R \end{bmatrix} \begin{bmatrix} {}^{A_i}a_i & \Lambda_i & 0 \\ 0 & -{}^{A_i}\hat{a}_i & \psi_{\omega_i} \end{bmatrix}, \tag{19}$$

with

$${}^{A_i}a_i \equiv {}^{A_i}\ddot{x}_{Ai} - {}^{A_i}G,$$

and

$$p_i = [m_i, m_i r_i^T, I_{pi}^T]^T.$$

Equation (19) is a linear representation of the dynamic equation. If a column of the dynamic matrix is composed of zeros only it can be cancelled together with the corresponding parameter, since it has no influence on the dynamics. Furthermore, if columns of the dynamic matrix can be

[1] index i is omitted in the following to simplify the notation

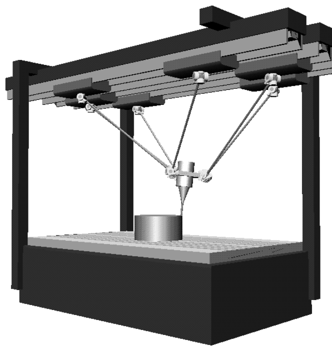

Fig. 1. Drawing of the Hexaglide parallel machine tool

expressed as linear combinations of other columns, the corresponding parameters can be grouped and only one column kept in the dynamic matrix (Gautier, 1990; Goldenberg *et al.*, 1992; Khalil and Bennis, 1995).

In nonlinear adaptive control, it is necessary to compute the inverse dynamic model in real-time. For fully parallel robots, an efficient solution is to neglect the inertias of the legs. It has been shown in (Codourey, 1991) that for most fully parallel robots this assumption leads to a simple model without too much loss. In this case, finding a linear form of the dynamics is highly simplified by using the method proposed in this paragraph. Indeed, the complicated kinematics of the legs does not need to be computed, and the robot is reduced to few bodies. As an illustration, the method described in this paragraph is applied to the Hexaglide, a 6 dof parallel machine.

4. MODELING OF THE HEXAGLIDE MACHINE

The Hexaglide is a concept developed at the Institute of Machine Tools of the Swiss Federal Institute of Technology Zurich (Wiegand *et al.*, 1996). It has a fully parallel structure with 6 dof and is intended to be used as a high speed milling machine. A drawing of the machine is shown in figure 1. It is similar to a Stewart platform but instead of variable length bars, it has constant length bars linking the platform to 6 linear motors distributed on 3 linear rails. In this section, we first derive the kinematics of the Hexaglide and then use it for calculating a linear form of the dynamic equation.

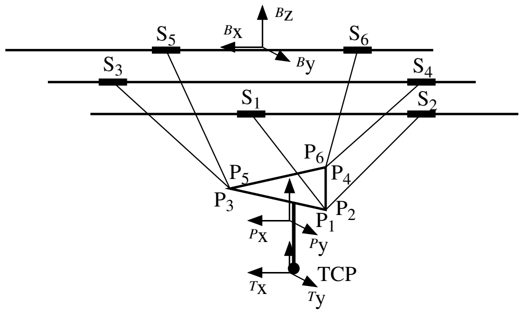

Fig. 2. Definition of the frames and geometric parameters of the Hexaglide

4.1 *Kinematics of the Hexaglide*

For modeling the Hexaglide structure, a base reference frame $\mathcal{B}$ is defined as shown in figure 2. A frame $\mathcal{P}$ is attached to the center of gravity of the platform. A third frame $\mathcal{T}$ is attached to the tool-center-point (TCP) of the robot. The points linking the legs to the platform are noted P_i, $i = 1, ..., 6$, and each leg is attached to the linear motor at the points S_i, $i = 1, ..., 6$. The pose of the TCP is represented by the vector

$$x_T \equiv [x, y, z, \alpha, \beta, \gamma]^T , \tag{20}$$

where x, y, z are the cartesian positions of the TCP and γ, β, α the fixed angles ZYX representation of its rotation (Craig, 1989). The rotation matrix between the frames $\mathcal{T}$ and $\mathcal{B}$ is given by:

$$^B_T R = R_x(\alpha) \cdot R_y(\beta) \cdot R_z(\gamma) \tag{21}$$

The inertia of the legs will be neglected in order to simplify the computation of the dynamic model. With this assumption, the robot can be considered as composed of 7 bodies, i.e. 6 motors and the platform.

In order to compute the dynamic equation, the velocity and acceleration of each body, as well as the Jacobian relating its cartesian velocity to the joint velocity of the robot, have to be known. In the following, the motors will be represented by the indices 1 to 6 and the platform by index 7. Let q_i denote the joint coordinates, and the joint vector be:

$$q \equiv [q_1, q_2, q_3, q_4, q_5, q_6]^T . \tag{22}$$

To simplify the notation, the Jacobian matrices of the motors J_{0i} can be decomposed in two parts: one part affecting linear velocities (J_{0vi}) and another affecting angular velocities only ($J_{0\omega i}$):

$$J_{0i} \equiv \begin{bmatrix} J_{0vi} \\ J_{0\omega i} \end{bmatrix} \tag{23}$$

We have

$$J_{0v1} = \begin{bmatrix} 1\,0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0\,0 \end{bmatrix},$$

$$\vdots$$

$$J_{0v6} = \begin{bmatrix} 0\,0\,0\,0\,0\,1 \\ 0\,0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0\,0 \end{bmatrix}, \tag{24}$$

and

$$\ddot{x}_1 = \begin{bmatrix} \ddot{q}_1 \\ 0 \\ 0 \end{bmatrix}, \quad \cdots, \quad \ddot{x}_6 = \begin{bmatrix} \ddot{q}_6 \\ 0 \\ 0 \end{bmatrix}. \tag{25}$$

Because no rotation occurs at the motor level, the corresponding Jacobian matrices, angular velocities and accelerations are all equal to zero:

$$J_{0\omega i} = 0\,, \tag{26}$$

$$\omega_i = 0\,, \quad \dot{\omega}_i = 0\,; \quad i = 1, ..., 6\,. \tag{27}$$

The Jacobian matrix J_T of the TCP relates the time derivative of the operational coordinates and the joints velocity vector:

$$\dot{x}_T \equiv J_T\,\dot{q}, \tag{28}$$

with x_T defined in (20) and q in (22).

The Jacobian J_T has been derived in (Honegger, 1996) using the partial differentiation of the inverse geometric model of the machine. J_T is representation dependant. In order to find the instantaneous linear and angular velocities of the TCP, J_T has to be multiplied by the representation dependant matrix $E^+(x_T)$ (Craig, 1989):

$$\dot{x}_{0T} = E^+\,J_T\,\dot{q} = J_{0T}\,\dot{q}\,, \tag{29}$$

with

$$\dot{x}_{0T} \equiv [v_{0T}^T, \omega_{0T}^T]^T,$$
$$E^+ \equiv \begin{bmatrix} 1_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & E_r^+ \end{bmatrix}$$

and

$$E_r^+ = \begin{bmatrix} 1 & 0 & \sin\beta \\ 0 & \cos\alpha & -\cos\beta\sin\alpha \\ 0 & \sin\alpha & \cos\beta\cos\alpha \end{bmatrix}. \tag{30}$$

It has to be noted that matrix E^+ is singular for $\beta = \pi/2$. This is however not a problem in our case, since this happens outside of the workspace of the robot.

From (29), it follows for the Jacobian matrix of the platform (body 7):

$$J_{07} = J_{0T} = E^+ J_T\,. \tag{31}$$

The linear acceleration of body 7 is given by $\ddot{x}_T$ which is calculated by numerical differentiation of $\dot{x}_T$. The angular velocity and acceleration are further calculated as follows:

$${}^B\omega_T = E_r^+ \begin{bmatrix} \dot{\alpha} \\ \dot{\beta} \\ \dot{\gamma} \end{bmatrix}, \tag{32}$$

$${}^B\dot{\omega}_T = \frac{d}{dt}({}^B\omega_T) \quad . \tag{33}$$

4.2 *Linear form of the dynamic equation*

Using equation (19), and noting that the use of equation (7) leads to the identity matrix for J_τ, the dynamic model of the Hexaglide can be written in a linear form of the dynamic parameters as follows:

$$\tau = \Psi\,p \tag{34}$$

where the matrix Ψ and correspondingly the parameter vector p can be splitted in three parts:

$$\Psi \equiv \left[\,\Psi_{1..6}\;\;\Psi_b\;\;\Psi_7\,\right], \tag{35}$$

$$p \equiv \left[\,p_{1..6}\;\;p_b\;\;p_7\,\right]^T. \tag{36}$$

In this representation, the index b corresponds to the friction, index 1..6 to the dynamics of the motors and 7 to the dynamics of the platform. We evaluate these terms in the next three sections.

4.2.1. *Determination of* $\Psi_{1..6}$ *and* $p_{1..6}$

The Jacobians $J_{0i}, i = 1, ..., 6$, have been calculated above (23, 24, 27). The Ω_i matrices are defined as follows:

$$\Omega_i \equiv \begin{bmatrix} {}^Ba_i & \Lambda_i & 0 \\ 0 & -{}^B\hat{a}_i & \psi_{\omega i} \end{bmatrix}, \quad i = 1, ..., 6, \tag{37}$$

where a is the linear acceleration of the body, $\hat{a}$ the cross product matrix form of a as defined in equation (16), and Λ and ψ as defined in equations (13) and (14).

Using equations (25) and (27), we can compute Ω_i:

$$\Omega_i = \begin{bmatrix} \begin{matrix} \ddot{q}_i \\ 0 \\ g \end{matrix} & 0_{3\times3} & 0_{3\times6} \\ 0_{3\times1} & \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & \ddot{q}_i \\ 0 & -\ddot{q}_i & 0 \end{matrix} & 0_{3\times6} \end{bmatrix} \tag{38}$$

We can now determine the elements $J_{0i}^T\Omega_i$ of the matrix $\Psi_{1..6}$:

$$J_{01}^T \Omega_1 = \begin{bmatrix} \ddot{q}_1 & \\ 0 & \\ 0 & 0_{6\times 9} \\ 0 & \\ 0 & \\ 0 & \end{bmatrix},$$

$$\vdots$$

$$J_{06}^T \Omega_6 = \begin{bmatrix} 0 & \\ 0 & \\ 0 & 0_{6\times 9} \\ 0 & \\ 0 & \\ \ddot{q}_6 & \end{bmatrix}. \qquad (39)$$

As expected, for the bodies 1 to 6, there is no contribution of inertial terms. These terms can be cancelled, reducing correspondingly the parameter vector. This gives:

$$\Psi_{1..6} \equiv \begin{bmatrix} \ddot{q}_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \ddot{q}_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \ddot{q}_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & \ddot{q}_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & \ddot{q}_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & \ddot{q}_6 \end{bmatrix}, \qquad (40)$$

$$p_{1..6} \equiv [m_1, m_2, ..., m_6]^T. \qquad (41)$$

4.2.2. *Friction terms*

For the Hexaglide, we will assume that the friction is located at the motorized joints and is composed of viscous and Coulomb friction:

$$b_i(\dot{q}_i) \equiv b_i \operatorname{sign}(\dot{q}_i) + c_i \dot{q}_i, \quad i = 1, ..., 6. \qquad (42)$$

Using the Jacobian J_{0i} of the motor defined in equations (23, 24, 27), we find

$$\Psi_b \equiv \begin{bmatrix} \operatorname{sign}(\dot{q}_1) & \dot{q}_1 & & 0 & 0 \\ 0 & 0 & & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & & 0 & 0 \\ 0 & 0 & & 0 & 0 \\ 0 & 0 & & \operatorname{sign}(\dot{q}_6) & \dot{q}_6 \end{bmatrix} \qquad (43)$$

and

$$p_b \equiv [b_1, c_1, ..., b_6, c_6]^T. \qquad (44)$$

4.2.3. *Determination of Ψ_7 and p_7*

For body 7 (platform), the parameters ($\ddot{x}_T$, ω_T, $\dot{\omega}_T$) needed to find Ω_7 have been calculated in equations (28), (32) and (33). We have:

$$\Omega_7 = \begin{bmatrix} {}^B a_T & {}^B_T R \, \Lambda_7 & 0 \\ 0 & -{}^B \hat{a}_T \, {}^B_T R & {}^B_T R \, \psi_{\omega 7} \end{bmatrix}, \qquad (45)$$

with ${}^B a_T = {}^B \ddot{x}_T - G$, $G = [0, 0, -g]^T$, and $\psi_{\omega 7}$ and Λ_7 calculated using the components of ${}^T\omega_T$ and ${}^T\dot{\omega}_T$ in frame $\mathcal{T}$. The Jacobian matrix J_{07} comes from equation (31). We obtain finally

$$\Psi_7 \equiv J_{07}^T \Omega_7 \qquad (46)$$

and

$$p_7 \equiv [m_7, m_7 r_{7,x}, m_7 r_{7,y}, m_7 r_{7,z}, \\ I_{7,xx}, I_{7,xy}, ..., I_{7,zz}]^T. \qquad (47)$$

4.2.4. *Complete form*

We finally find

$$\tau = \Psi \, p \equiv \begin{bmatrix} \Psi_{1..6} & \Psi_b & \Psi_7 \end{bmatrix} \begin{bmatrix} p_{1..6} \\ p_b \\ p_7 \end{bmatrix} \qquad (48)$$

with $\Psi_{1..6}$ and $p_{1..6}$ defined by equations (40, 41), Ψ_b and p_b in (43, 44), and Ψ_7 and p_7 in (46, 47). This relatively compact dynamic equation can be used for nonlinear control of the Hexaglide. This will be shown in the next section.

5. NONLINEAR ADAPTIVE CONTROL

Algorithms from nonlinear adaptive control theory based on the minimization of the tracking error function (Craig, 1988; Slotine and Li, 1987; Sadegh and Horowitz, 1990) provide the nonlinear control and the dynamical calibration simultaneously. In the following, we will use the Adaptive FeedForward Controller (AFFC) of (Sadegh and Horowitz, 1990).

The control law of the AFFC is (figure 3):

$$\tau = \Psi(q_d, \dot{q}_d, \ddot{q}_d) \, p + \tau_{FB}, \qquad (49)$$

where τ_{FB} is the joint linear feedback portion of the controller:

$$\tau_{FB} \equiv K_p \, e + K_d \, \dot{e} \qquad (50)$$

where $e \equiv q_d - q$ is the error vector between the desired and actual joint positions and K_p and K_d are diagonal matrices with positive terms.

In the AFFC, the adaptation is performed by gradient descent of the tracking error function

$$E = (\tau - \Psi \, p)^2, \qquad (51)$$

which gives the *learning rule*

$$\widetilde{p}_{new} \equiv \widetilde{p}_{old} + \Delta p, \quad \Delta p \equiv \Gamma \, \Psi^T \, \tau_{FB}, \qquad (52)$$

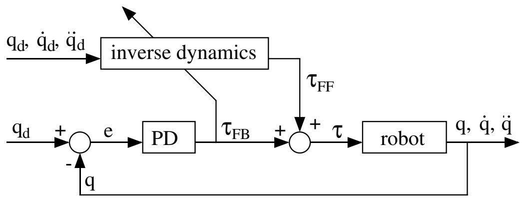

Fig. 3. Functional scheme of the *adaptative feedforward controller*.

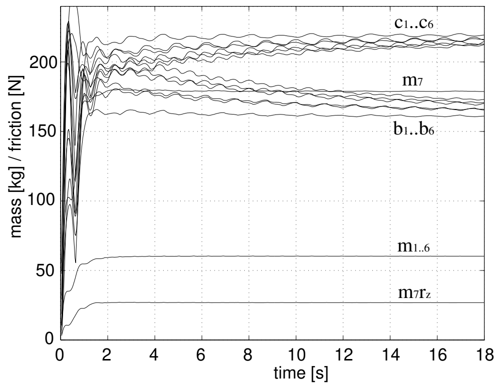

Fig. 4. Learning the friction and masses using a translational movement

with Γ a positive definite matrix composed of the *learning factors*.

6. SIMULATION RESULTS

This section presents some simulations performed in order to test the nonlinear adaptive controller approach designed for the Hexaglide.

To simplify the learning of the parameters, the six masses of the sliders which should all be equal are collected together in a single parameter $m_{1..6}$. Because they are more subject to variation, the friction terms are however considered individually. The frame attached to the TCP is supposed to be oriented such that the inertia moments $I_{7,xy}$, $I_{7,xz}$ and $I_{7,yz}$ are equal to zero. We further suppose that $r_7 = [0, 0, r_z]^T$, i.e. that only the position of the center of mass in the z direction of frame $\mathcal{T}$ has to be learned. Thus, the matrix Ψ and the parameter vector p can be rewritten as

$$\Psi \equiv \begin{bmatrix} \ddot{q}_1 & & \\ \ddot{q}_2 & & \\ \ddot{q}_3 & \Psi_b & J_{07}^T \Omega_7 \\ \ddot{q}_4 & & \\ \ddot{q}_5 & & \\ \ddot{q}_6 & & \end{bmatrix}, \tag{53}$$

with Ψ_b and $J_{07}^T \Omega_7$ defined in equation (43) and (46) respectively, and

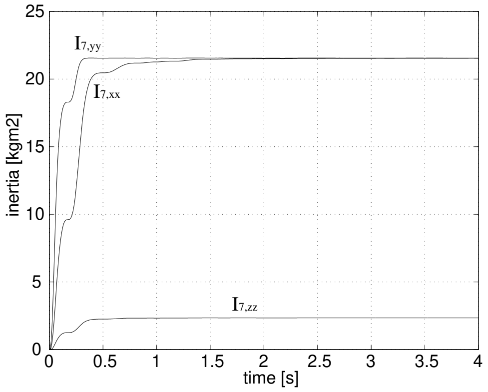

Fig. 5. Learning the inertia tensor by performing pure rotations

	$m_{1..6}$ [kg]	$b_{1..6}$ (min/max) [N]	$c_{1..6}$ (min/max) [N]	m_7 [kg]
actual	60.00	160.0	220	180.0
learned	60.27	161.04/173.09	212.5/219.38	178.73
	$m_7 r_z$ [kgm]	$I_{7,xx}$ [kgm2]	$I_{7,yy}$ [kgm2]	$I_{7,zz}$ [kgm2]
actual	27.0	21.547	21.547	2.340
learned	26.88	21.535	21.553	2.343

Table 1. Comparison of learned with actual parameters

$$p \equiv [m_{1..6}, b_1, c_1, ..., b_6, c_6, m_7, m_7 r_z, I_{7,xx}, I_{7,yy}, I_{7,zz}]^T. \tag{54}$$

Figures 4 and 5 show how the parameters have been learned.

Table 1 compares the learned parameters with their real value. In this computer-simulation the dynamic parameters are learned in a relatively short time and with a good precision. It has however to be mentioned that a simplified model of the dynamics of the machine has been used, including however viscous and Coulomb friction terms. Furthermore, no other perturbations such as noise in the measurement data have been used. Because such effects are inherent to a real application, the learning process won't probably be so fast. This simulation presents therefore only an optimal case, but demonstrates the possibility to apply nonlinear adaptive control to complex parallel robots.

7. CONCLUSION

Finding a linear form of the dynamic equation of parallel robots is a difficult task, especially when a real-time implementation is needed. In this paper, we presented a new method based on the virtual work principle. It considers each body separately in a body-oriented approach. Jacobian matrices are used to project the forces and torques of each body onto the space of actuated joints. An advantage of the method is that simplifying hypothesis can be used at the very beginning already. As an example, the method was applied to the Hexaglide, a 6 dof fully parallel platform. It led to compact equations which can be used in a nonlinear adaptive control scheme. Simulation results are presented demonstrating the possibility to apply nonlinear adaptive control to complex parallel robots.

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