Adaptive Control of the Hexaglide, a 6 dof Parallel Manipulator

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Abstract

This paper presents the dynamic equation, non-linear control and dynamic parameters identification of the Hexaglide, a new 6 dof parallel manipulator intended to be used as a high speed milling machine.

Using a method based on the virtual work principle, the dynamic equation is found in a compact linear form and then used in a non-linear adaptive control algorithm based on the minimization of the tracking error. The dynamic parameters are learned during motion and introduced in an inverse dynamic model used as a feedforward compensator. Specific trajectories, exciting the parameters separately, enable a fast stepwise learning of the 12 parameters. A simulation demonstrates the validity of the approach.

1 Introduction

The Hexaglide is a concept developed at the Institute of Machine Tools of the Swiss Federal Institute of Technology Zurich [16]. The machine is a fully parallel structure with 6 dof intended to be used as a high speed milling machine. A sketch of the machine is shown in figure 1. It is similar to a Stewart platform but instead of variable length bars, it has constant length bars (2) linking the platform (1) to 6 linear motors (3) distributed on 3 linear rails (4).

The advantages of parallel manipulators over serial ones are well known. The most important is certainly the possibility to keep the motors fixed on the base. This has the consequence that light limbs can be used, and fast movements can be performed. Besides, parallel manipulators have a high stiffness and are thus well suited for fast and accurate manipulation.

To perform accurate movements, a good control is however required. Due to the closed mechanical chains, strong interactions between the different motors appear. This, combined with high speed motion makes the dynamics of parallel manipulators highly nonlinear. To minimise the tracking error, a dynamics feedforward compensator has to be used together with a linear feedback controller.

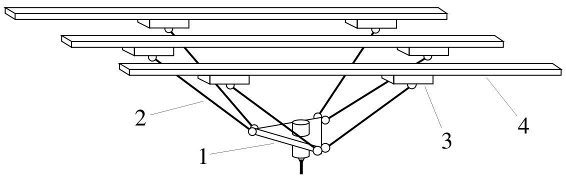

Figure 1: Sketch of the Hexaglide

In order to perform a suitable compensation, the parameters of the dynamic equation of the robot must be known exactly. Several methods can be used to determine these parameters. They are usually issued from the CAD drawings, or the manipulator is disassembled and the parameters of each body are measured. A simple non-invasive way to identify the parameters is to use nonlinear adaptive control. This leads to more accurate parameters [3], can be performed on-line without modifying the control structure, and varying parameters such as friction can continuously be updated.

In order to apply algorithms from nonlinear adaptive control theory, it is necessary to write the dynamic equation in a linear function of parameters. In section 2, we use a method based on the virtual work principle [4] to find the dynamic equation of the Hexaglide as a function of 12 dynamic parameters. As it is difficult to identify these 12 parameters in one step, we study in section 3 how they can be learned stepwise, by minimising the tracking error along specific trajectories.

2 Equations of motion

In this section, we first derive the kinematics of the Hexaglide and then use it for calculating a linear form of the dynamic equation.

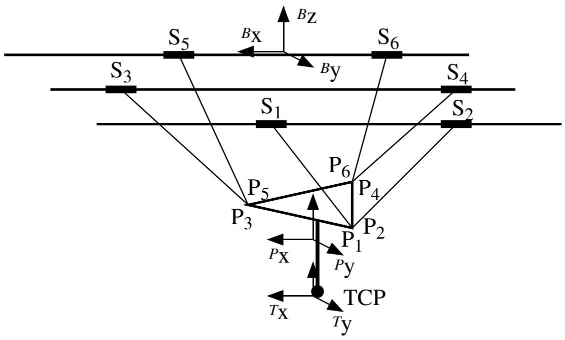

Figure 2: Definition of the frames and geometric parameters of the Hexaglide

2.1 Kinematics of the Hexaglide

For modeling the Hexaglide structure, a base reference frame $\mathcal{B}$ is defined as shown in figure 2. A frame $\mathcal{P}$ is attached to the center of gravity of the platform. A third frame $\mathcal{T}$ is attached to the tool-center-point (TCP) of the robot. The points linking the legs to the platform are noted P_i, $i = 1, ..., 6$, and each leg is attached to the linear motor at the points S_i, $i = 1, ..., 6$. The pose of the TCP is represented by the vector

$$x_T \equiv [x, y, z, \alpha, \beta, \gamma]^T \,, \tag{1}$$

where x, y, z are the cartesian positions of the TCP and α, β, γ the fixed angles ZYX representation of its rotation [6]. The rotation matrix between the frames $\mathcal{T}$ and $\mathcal{B}$ is given by:

$${}^B_T R = R_x(\alpha) \cdot R_y(\beta) \cdot R_z(\gamma). \tag{2}$$

The inertia of the legs will be neglected in order to simplify the computation of the dynamic model. With this assumption, the robot can be considered as composed of 7 bodies, i.e. 6 motors and the platform.

In order to compute the dynamic equation, the velocity and acceleration of each body, as well as the Jacobian relating its cartesian velocity to the joint velocity of the robot, have to be known. In the following, the motors will be represented by the indices 1 to 6 and the platform by index 7. Let q_i denote the joint coordinates, and the joint vector be:

$$q \equiv [q_1, q_2, q_3, q_4, q_5, q_6]^T \,. \tag{3}$$

To simplify the notation, the Jacobian matrices of the motors J_{0i} can be decomposed in two parts: one part affecting linear velocities (J_{0vi}) and another affecting angular velocities only ($J_{0\omega i}$):

$$J_{0i} \equiv \begin{bmatrix} J_{0vi} \\ J_{0\omega i} \end{bmatrix} \tag{4}$$

We have

$$J_{0v1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\vdots$$

$$J_{0v6} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{5}$$

and

$$\ddot{x}_1 = \begin{bmatrix} \ddot{q}_1 \\ 0 \\ 0 \end{bmatrix}, \quad \ddot{x}_2 = \begin{bmatrix} \ddot{q}_2 \\ 0 \\ 0 \end{bmatrix}, \quad \cdots, \quad \ddot{x}_6 = \begin{bmatrix} \ddot{q}_6 \\ 0 \\ 0 \end{bmatrix}. \tag{6}$$

Because no rotation occurs at the motor level, the corresponding Jacobian matrices, angular velocities and accelerations are all equal to zero:

$$J_{0\omega i} = 0\,, \quad \omega_i = 0\,, \quad \dot{\omega}_i = 0\,; \quad i = 1, ..., 6\,. \tag{7}$$

The Jacobian matrix J_T of the TCP relates the time derivative of the operational coordinates and the joints velocity vector:

$$\dot{x}_T \equiv J_T\, \dot{q}, \tag{8}$$

with x_T defined in (1) and q in (3).

The Jacobian J_T has been derived in [7] using the partial differentiation of the inverse geometric model of the machine. J_T is representation dependant. In order to find the instantaneous linear and angular velocities of the TCP, it has to be multiplied by the representation dependant matrix $E^+(x_T)$ [9]:

$$\dot{x}_{0T} = E^+\, J_T\, \dot{q} = J_{0T}\, \dot{q}\,, \tag{9}$$

with

$$\begin{aligned} \dot{x}_{0T} &\equiv [v_{0T}^T, \omega_{0T}^T]^T, \\ E^+ &\equiv \begin{bmatrix} E_p^+ & 0_{3\times3} \\ 0_{3\times3} & E_r^+ \end{bmatrix}. \end{aligned}$$

In our case

$$E_p^+ = 1_{3\times3} \quad ,$$

and

$$E_r^+ = \begin{bmatrix} 1 & 0 & \sin\beta \\ 0 & \cos\alpha & -\cos\beta\sin\alpha \\ 0 & \sin\alpha & \cos\beta\cos\alpha \end{bmatrix}. \tag{10}$$

From (9), it follows for the Jacobian matrix of the platform (body 7):

$$J_{07} = J_{0T} = E^+ J_T\,. \tag{11}$$

The linear acceleration of body 7 is given by $\ddot{x}_T$ which is calculated by numerical differentiation of $\dot{x}_T$. The angular velocity and acceleration are further calculated as follows:

$$
\begin{aligned}
{}^B\omega_T &= E_r^+ \begin{bmatrix} \dot{\alpha} \\ \dot{\beta} \\ \dot{\gamma} \end{bmatrix}, && (12) \\
{}^B\dot{\omega}_T &= \frac{d}{dt}({}^B\omega_T) \quad . && (13)
\end{aligned}
$$

2.2 Linear form of the dynamic equation

Based on the virtual work principle, we have developed a body-oriented method for deriving the dynamic equation of a system of rigid bodies in a linear form as required for realising a dynamic calibration [4]. The resulting equation has the form

$$
\begin{aligned}
\tau &= \Psi\, p && (14) \\
&= J_\tau^{-1} \left[\begin{array}{cccc} J_{01}^T\Omega_1 & J_{02}^T\Omega_2 & \dots & J_{0N}^T\Omega_N \end{array} \right] p.
\end{aligned}
$$

In this equation, $p \equiv [p_1, p_2, ..., p_N]^T$ is the vector of dynamic parameters. J_τ is used to extract the actuator forces [4] and can be determined using

$$J_\tau = \left[\begin{array}{cccc} J_{01}^T\Upsilon_1 & J_{02}^T\Upsilon_2 & \cdots & J_{06}^T\Upsilon_6 \end{array} \right]. \tag{15}$$

Since all the motors are linear motors acting in the direction of axis x of their respective frame, $\Upsilon_i \equiv [1,0,0,0,0,0]^T$. It finally turns out that J_τ is the identity matrix:

$$J_\tau = 1_{6\times 6}. \tag{16}$$

In equation (14), the matrix Ψ and correspondingly the parameter vector p can be splitted in three parts:

$$\Psi \equiv \left[\begin{array}{ccc} \Psi_{1..6} & \Psi_b & \Psi_7 \end{array} \right], \quad p \equiv \left[\begin{array}{ccc} p_{1..6} & p_b & p_7 \end{array} \right]^T. \tag{17}$$

In this representation, the index b corresponds to the friction, index 1..6 to the dynamics of the motors and 7 to the dynamics of the platform. We will now evaluate these three terms.

2.2.1 Determination of $\Psi_{1..6}$ and $p_{1..6}$

The Jacobians $J_{0i}, i = 1, ..., 6$, have been calculated above (4, 5, 7). The Ω_i matrices are defined as follows:

$$\Omega_i \equiv \begin{bmatrix} {}^Ba_i & \Lambda_i & 0 \\ 0 & -{}^B\hat{a}_i & \psi_{\omega i} \end{bmatrix}, \quad i = 1, ..., 6, \tag{18}$$

where a is the linear acceleration of the body. Λ and ψ are defined as

$$\Lambda =^B \hat{\omega}_i \, {}^B\hat{\omega}_i +^B \hat{\dot{\omega}}_i, \tag{19}$$

$$\psi_\omega =^B \hat{\omega}_i \, {}^B\bar{\omega}_i + {}^B\bar{\dot{\omega}}_i, \tag{20}$$

where ˆ means the skew symmetric matrix corresponding to the cross product [6] and

$$\bar{\omega} = \begin{bmatrix} \omega_x & \omega_y & \omega_z & 0 & 0 & 0 \\ 0 & \omega_x & 0 & \omega_y & \omega_z & 0 \\ 0 & 0 & \omega_x & 0 & \omega_y & \omega_z \end{bmatrix}. \tag{21}$$

Using equations (6) and (7), we can compute Ω_i:

$$\Omega_i = \begin{bmatrix} \begin{matrix} \ddot{q}_i \\ 0 \\ g \end{matrix} & 0_{3\times 3} & 0_{3\times 6} \\ 0_{3\times 1} & \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & \ddot{q}_i \\ 0 & -\ddot{q}_i & 0 \end{matrix} & 0_{3\times 6} \end{bmatrix} \tag{22}$$

We can now determine the elements $J_{0i}^T\Omega_i$ of the matrix $\Psi_{1..6}$:

$$J_{01}^T\Omega_1 = \begin{bmatrix} \begin{matrix} \ddot{q}_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{matrix} & 0_{6\times 9} \end{bmatrix}, \tag{23}$$

and so on for $i = 2$ to 6. As expected, for bodies 1 to 6, there is no contribution of inertial terms. These terms can be cancelled, reducing correspondingly the parameter vector. This gives finally:

$$
\begin{aligned}
\Psi_{1..6} &\equiv \begin{bmatrix} \ddot{q}_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \ddot{q}_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \ddot{q}_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & \ddot{q}_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & \ddot{q}_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & \ddot{q}_6 \end{bmatrix}, && (24) \\
p_{1..6} &\equiv [m_1, m_2, ..., m_6]^T. && (25)
\end{aligned}
$$

2.2.2 Friction terms

For the Hexaglide, we will assume that the friction is located at the motorized joints and is composed of Coulomb friction only:

$$b_i(\dot{q}_i) \equiv b_i \, \mathrm{sign}(\dot{q}_i), \quad i = 1, ..., 6. \tag{26}$$

Using the motor Jacobian J_{0i} defined in equations (4, 5, 7), we find

$$\Psi_b \equiv \begin{bmatrix} \mathrm{sign}(\dot{q}_1) & 0 & & 0 \\ 0 & \mathrm{sign}(\dot{q}_2) & & 0 \\ 0 & 0 & \cdots & 0 \\ 0 & 0 & & 0 \\ 0 & 0 & & 0 \\ 0 & 0 & & \mathrm{sign}(\dot{q}_6) \end{bmatrix} \tag{27}$$

and

$$p_b \equiv [b_1, b_2, ..., b_6]^T. \tag{28}$$

2.2.3 Determination of Ψ_7 and p_7

For body 7 (platform), the parameters ($\ddot{x}_T$, ω_T, $\dot{\omega}_T$) needed to find Ω_7 have been calculated in equations (8), (12) and (13). We have:

$$\Omega_7 = \begin{bmatrix} {}^B a_T & {}^B_T R\, \Lambda_7 & 0 \\ 0 & -{}^B\hat{a}_T\, {}^B_T R & {}^B_T R\, \psi_{\omega 7} \end{bmatrix}, \tag{29}$$

with ${}^B a_T = {}^B\ddot{x}_T - G$, $G = [0, 0, -g]^T$, and $\psi_{\omega 7}$ and Λ_7 calculated using the components of ${}^T\omega_T$ and ${}^T\dot{\omega}_T$ in frame $\mathcal{T}$. The Jacobian matrix J_{07} comes from equation (11). We obtain finally

$$\Psi_7 \equiv J_{07}^T \Omega_7 \tag{30}$$

and

$$p_7 \equiv [m_7, m_7 r_{7,x}, m_7 r_{7,y}, m_7 r_{7,z}, I_{7,xx}, I_{7,xy}, ..., I_{7,zz}]^T. \tag{31}$$

2.2.4 Complete form

We finally find the following dynamic equation:

$$\tau = \Psi\, p \equiv \begin{bmatrix} \Psi_{1..6} & \Psi_b & \Psi_7 \end{bmatrix} \begin{bmatrix} p_{1..6} \\ p_b \\ p_7 \end{bmatrix} \tag{32}$$

with $\Psi_{1..6}$ and $p_{1..6}$ defined by equations (24, 25), Ψ_b and p_b in (27, 28), and Ψ_7 and p_7 in (30, 31). This relatively compact dynamic equation can be used for nonlinear control of the Hexaglide. Using its linearity, the parameters p can also be identified during the movements. This will be shown in the next section.

3 Nonlinear adaptive control

Algorithms from nonlinear adaptive control theory based on the minimization of the tracking error function [5, 13, 12] provide the nonlinear control and the dynamical calibration simultaneously.

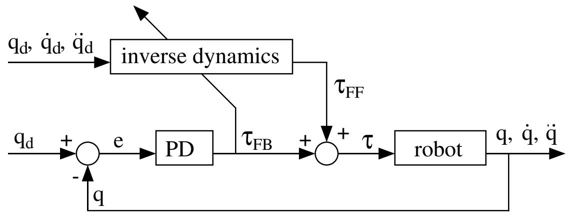

Figure 3: Functional scheme of the *adaptative feedforward controller.*

The calibration requires only a few addidional computations and can generally be performed on-line.

In the following, we will use the Adaptive FeedForward Controller [12], because it is simpler to implement than other algorithms [14, 3], is robust to noise [12], and leads to similar or better results than its competitors [10].

3.1 The Adaptive FeedForward Controller

The control law of the AFFC is:

$$\tau = \Psi(q_d, \dot{q}_d, \ddot{q}_d)\, p + \tau_{FB}, \tag{33}$$

where τ_{FB} is the joint linear feedback portion of the controller:

$$\tau_{FB} \equiv K_p\, e + K_d\, \dot{e} \tag{34}$$

where $e \equiv q_d - q$ is the error vector between the desired and actual joint positions and K_p and K_d are diagonal matrices with positive terms.

Let us suppose that the parameters composing p are totally unknown. As an initial value, we set[1] $\widetilde{p} \equiv 0$. At the beginning the robot is thus fully controlled with the linear joint feedback controller. In the adaptive paradigm, the parameters will be learned during the motion. After adaptation, the correct p (i.e. the inverse dynamics) should have been learned. In this case the linear controller is used only to correct the unmodelled dynamics.

In the AFFC, the adaptation is performed by gradient descent of the tracking error function

$$E = (\tau - \Psi\, p)^2, \tag{35}$$

which gives the *learning rule*

$$\widetilde{p}_{new} \equiv \widetilde{p}_{old} + \Delta p, \quad \Delta p \equiv \Gamma\, \Psi^T\, \tau_{FB}, \tag{36}$$

with Γ a positive definite matrix composed of the *learning factors*.

[1] $\widetilde{\ }$ over a variable indicates that this variable is an estimation of the real one.

3.2 Simulation results

This section presents some simulations performed in order to test the nonlinear adaptive controller approach designed for the Hexaglide.

For the simulation, a direct dynamic model has been established [7]. In this model the masses of the bars are splitted into 2 parts added to the mass of the platform and to the mass of the corresponding motor respectively. The simulation model has further been extended to take into account the time delays of the electric drives.

The control scheme used is equivalent to that of figure 3. It consists of linear joint PD-controllers for the six actuators and a feedforward term composed of the inverse dynamic model of the robot (equ. (33)). The parameters of the dynamic model are adapted with the learning rule (36).

To simplify the learning of the parameters, the six masses of the sliders which should all be equal are collected together in a single parameter $m_{1..6}$. Because they are more subject to variation, the friction terms are however considered individually. The frame attached to the TCP is supposed to be oriented such that the inertia moments $I_{7,xy}$, $I_{7,xz}$ and $I_{7,yz}$ are equal to zero. We further suppose that $r_7 = [0, 0, r_z]^T$, i.e. that only the position of the center of mass in the z direction of frame $\mathcal{T}$ has to be learned. Thus, the matrix Ψ and the parameter vector p can be rewritten as

$$\Psi \equiv \begin{bmatrix} \ddot{q}_1 & & \\ \ddot{q}_2 & & \\ \ddot{q}_3 & \Psi_b & J_{07}^T \Omega_7 \\ \ddot{q}_4 & & \\ \ddot{q}_5 & & \\ \ddot{q}_6 & & \end{bmatrix}, \tag{37}$$

with Ψ_b and $J_{07}^T \Omega_7$ defined in equation (27) and (30) respectively, and

$$p \equiv [m_{1..6}, b_1, b_2, ..., b_6, m_7, m_7 r_z, I_{7,xx}, I_{7,yy}, I_{7,zz}]^T . \tag{38}$$

The mass and friction parameters of the motors and the mass of the platform can be learned by moving the tool in a strictly translational manner. Because the first moment $m_7 r_z$ is also excited by a translational motion of the platform, it has to be learned together with masses and friction terms. Figure 4 shows how the concerned parameters have been learned with a simple translational movement.

After this, the only parameters that remain to be learned are the inertia parameters $I_{7,xx}$, $I_{7,yy}$ and $I_{7,zz}$. Each of these parameters is learned by performing a rotation about the corresponding axis. Figure 5 shows how these parameters have been learned.

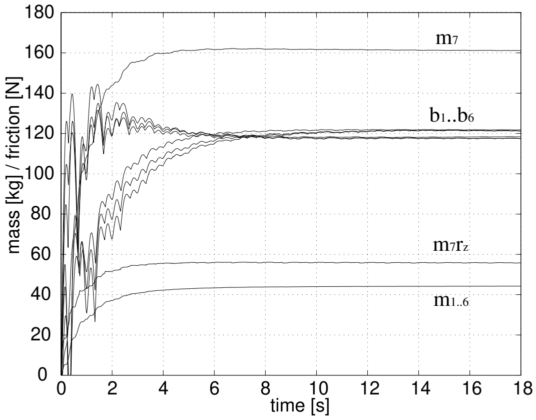

Figure 4: Learning the friction and masses using a translational movement

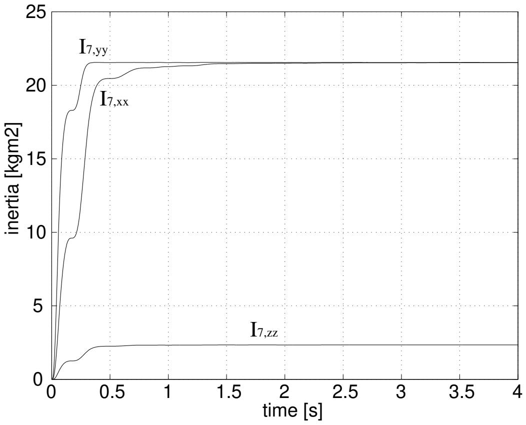

Figure 5: Learning the inertia tensor by performing pure rotations

Table 1 compares the learned parameters with their real value. In this computer-simulation the dynamic parameters are learned in a relatively short time and with a good precision. It has however to be mentioned that a simplified model of the dynamics of the machine has been used, with a simple Coulomb friction model. Furthermore, no other perturbation such as noise in the measurement data has been used. Because such effects are inherent to a real application, the learning process won't probably be so fast [3]. This simulation presents therefore only an optimal case, but demonstrates the possibility to apply nonlinear adaptive control to complex parallel robots.

	$m_{1..6}$ [kg]	$b_{1..6}$ (min) [N]	$b_{1..6}$ (max) [N]	m_7 [kg]
actual	45.00	120.0		160.0
learned	44.26	117.5	121.8	161.1
	$m_7 r_z$ [kgm]	$I_{7,xx}$ [kgm2]	$I_{7,yy}$ [kgm2]	$I_{7,zz}$ [kgm2]
actual	55.68	21.547	21.547	2.340
learned	55.81	21.535	21.553	2.343

Table 1: Comparison of learned with actual parameters

4 Conclusions

Using the Hexaglide as a fast and high precision milling machine requires to cancel the large dynamical effects as exactly as possible. In this paper, we derived the dynamical equation and proposed a method for identifying its many parameters using nonlinear adaptive control algorithms with a stepwise learning. A simulation showed that the learning is fast and accurate, and the resulting control is precise.

The proposed approach can not only be applied to fully parallel robots but also to other complex mechanical structures.

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