Robustness of Self-Sensing Magnetic Bearing

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ABSTRACT

Self-sensing magnetic bearings work without position sensors. The position measurement, which is required by the controller, can be deduced from the electromagnetic interaction between stator and rotor. The self-sensing method discussed in this paper is based on controlling the voltage over the coils and measuring the current through the coils. This method can be realized with a minimal amount of hardware and therefore with low costs. On the other hand several earlier experiments have shown, that this method needs a complicated adjustment procedure of the controller. Furthermore with optimum adjustment the system robustness is low.

The goal of this paper is to analyze the sensitivity of a self-sensing magnetic bearing to the controller parameters. It will be shown, that stability within the whole air gap can only be achieved at the expense of system robustness. The low system robustness is characterized by one or two low frequency poles of the closed-loop.

INTRODUCTION

A self-sensing (sensorless) magnetic bearing is a special case of a magnetic bearing, which needs no additional position sensors. The position information is deduced from the electromagnetic interaction between stator and rotor. The main advantage of self-sensing magnetic bearings is the reduction of the manufacturing costs. Furthermore they have properties, making them attractive for specific applications. The elimination of the position sensors enables a simple and reliable construction without sensor housing and cables. In addition such a mechanically optimized construction enables to design a more rigid rotor with higher natural frequencies.

Two essential methods are known for the self-sensing operation. One is a *self-sensing magnetic bearing with a linear controller*. This method extracts the position information from the coils currents. The controller is adjusting the voltage over the coils and stabilizes the levitation of the rotor. Such configuration leads to low robustness and low disturbance rejection, and therefore suits only for a specific class of applications, where low system costs are the primary design goal. A self-sensing magnetic bearing with linear control was first published by (Vischer, 1988). Subsequently the practical implementation was made and reported by (Colotti

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and Kucera, 1991) and the corresponding apparatus has been patented (Vischer et al., 1991). In (Müller et al., 1996) all radial degrees of freedom of a turbomolecular pump were stabilized with a self-sensing control. Additional publications with this subject are (Vischer and Bleuler, 1993; Mizuno, Namiki and Araki, 1996; Kucera, 1997). In patent (McCormack, 1992) a magnetic bearing rotor system is documented, which combines a self-sensing controller with a modulation setup. The modulation is used to remove the negative stiffness of the self-sensing magnetic bearing.

The *self-sensing magnetic bearing with modulation method* is based on generating a position signal from the air gap dependence of the coil impedance. The manufacturing costs of this method are higher, but the disturbance rejection and overall performance are improved (Kucera, 1997). Additional publications and patents on this subject can be found in (Scheffer and Guse, 1977; Okada, Matsumura and Nagai, 1992; Noh and Maslen, 1995, 1996a, 1996b; Mizuno, Namiki and Araki, 1996; Mizuno, Namiki and Araki, 1996; Rubner and Lindenau, 1996).

Earlier experimental setups proved the principal operation of different self-sensing magnetic bearings. But all of these setups showed low robustness and very difficult start up procedure arriving to an initial stable levitation. The goal of the present work is to solve the problem of the start up phase by a controller with an adjustable structure. The magnetic bearing has to be modeled with taking into consideration the leakage inductance (the part of the inductance which is not a function of the air gap) and the resistance of the coil. Based on an analytical model the parameter sensitivities are computed and used to adjust the controller in an optimized way.

NONLINEAR MODEL

The following nonlinear model is developed based on a differential configuration and is depicted in figure 1. Two electromagnets produce contractile forces F_1 and F_2, which are acting on the rotor. Other forces like gravitation, unbalance etc. are considered as a total disturbance force F_s. The voltages over the coils are u_1 and u_2, and the currents trough the coils are i_1 and i_2. The distances x_1 and x_2 are corresponding with the air gaps between the electromagnets and the rotor. The sum of the two air gaps equals $2x_0$, where x_0 is the nominal air gap at the equilibrium position of the rotor. The displacement x corresponds to the deviation of the rotor from the equilibrium position.

The most important properties of magnetic bearings can be described by equations (1)-(3). Equation (1) expresses the forces, which are produced by the electromagnets and the electrical function is given with equation (2) and is including the coil resistance R and the leakage inductance L_s.

$$F_1 = \frac{K}{4}\left(\frac{i_1}{x_1}\right)^2 \qquad F_2 = \frac{K}{4}\left(\frac{i_2}{x_2}\right)^2 \tag{1}$$

$$u_1 = Ri_1 + L_s\frac{di_1}{dt} + \frac{K}{2}\frac{d}{dt}\left(\frac{i_1}{x_1}\right) \qquad u_2 = Ri_2 + L_s\frac{di_2}{dt} + \frac{K}{2}\frac{d}{dt}\left(\frac{i_2}{x_2}\right) \tag{2}$$

$K = \mu_0 N^2 A$ is constant with N: turns of the coil, A: area of the core and μ_0: permeability of air. The equation of motion is

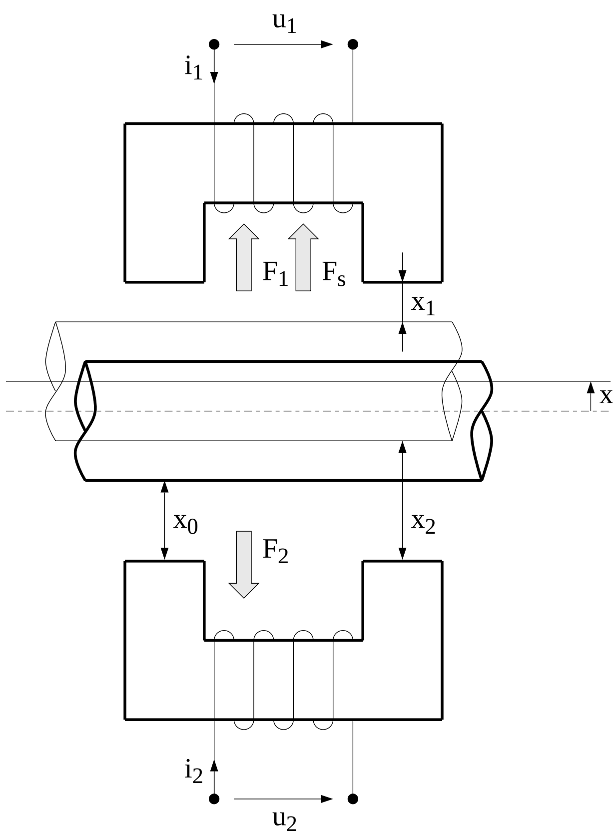

Figure 1.

$$m\frac{d^2x}{dt^2} = F_1 - F_2 + F_s \qquad (3)$$

where m is the rotor mass.

LINEARIZED MODEL

Electromagnets are biased with a current i_0. The bias voltage due to the coil resistance R is $u_0 = R\, i_0$. The deviations from the equilibrium values x_0, i_0 and u_0 are x, i and u and can be expressed as follows:

$$x_1 = x_0 - x \qquad x_2 = x_0 + x \qquad (4)$$

$$i_1 = i_0 + i \qquad i_2 = i_0 - i \qquad (5)$$

$$u_1 = u_0 + u \qquad u_2 = u_0 - u \qquad (6)$$

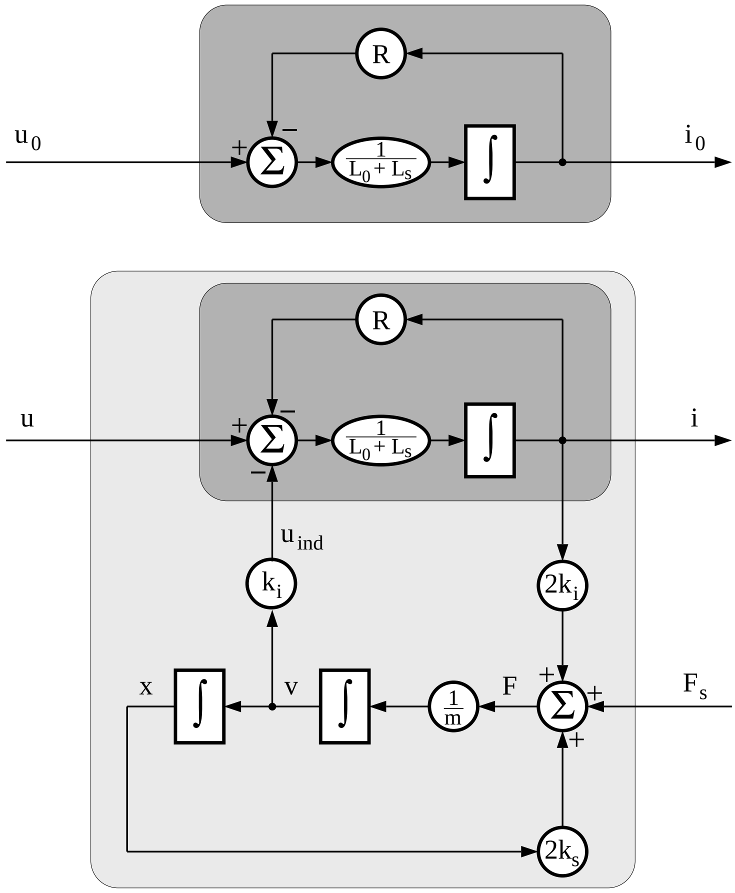

Figure 2.

Equations (1) and (2) can be linearized using equations (4)-(6),. A magnetic bearing with a differential configuration like in figure 1 leads to a 4th order MIMO[1] plant. By defining the system states as x, v, i and i_0 the MIMO plant can be divided into two SISO[2] plants with 1st and 3rd order. The block diagram of these SISO plants are shown in figure 2 and can be formulated by equations (7) and (8).

$$\frac{d}{dt}\begin{bmatrix} x \\ v \\ i \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{2k_s}{m} & 0 & \frac{2k_i}{m} \\ 0 & \frac{-k_i}{L_0 + L_s} & \frac{-R}{L_0 + L_s} \end{bmatrix}\begin{bmatrix} x \\ v \\ i \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L_0 + L_s} \end{bmatrix} u + \begin{bmatrix} 0 \\ \frac{1}{m} \\ 0 \end{bmatrix} F_s \qquad (7)$$

$$\frac{d}{dt} i_0 = \frac{-R}{L_0 + L_s} i_0 + \frac{1}{L_0 + L_s} u_0 \qquad (8)$$

1. Multiple Input Multiple Output
2. Single Input Single Output

where $k_s = \frac{K}{2}\frac{i_0^2}{x_0^3}$, $k_i = \frac{K}{2}\frac{i_0}{x_0^2}$ and $L_0 = \frac{K}{2x_0}$. The constant $\omega_p = \sqrt{\frac{2k_s}{m}}$ represents the open-loop dynamics of the magnetic bearing. The variable i_0 is the single state of equation (8). Close to the equilibrium position this variable doesn't change much and thus can be used for calculating k_s and k_i.

Using Laplace-transform, equations (7) and (8) can be expressed in the frequency domain. The transfer functions G_i and G_{i0} are written as

$$G_i(s) = \frac{i(s)}{u(s)} = \frac{s^2 - \frac{2k_s}{m}}{s^3(L_0 + L_s) + s^2 R - sL_s\frac{2k_s}{m} - R\frac{2k_s}{m}} \tag{9}$$

$$G_{i_0}(s) = \frac{i_0(s)}{u_0(s)} = \frac{1}{s(L_0 + L_s) + R} \tag{10}$$

The block diagram on the dark backgrounds of figure 2 consist of the electric subsystems (coils). The block diagram on the light background represents the electromechanical subsystem. The interaction between these two subsystems is made by the current i and the induced voltage u_{ind}. The rotor movement induces voltage over the coils which generates measurable current changes.

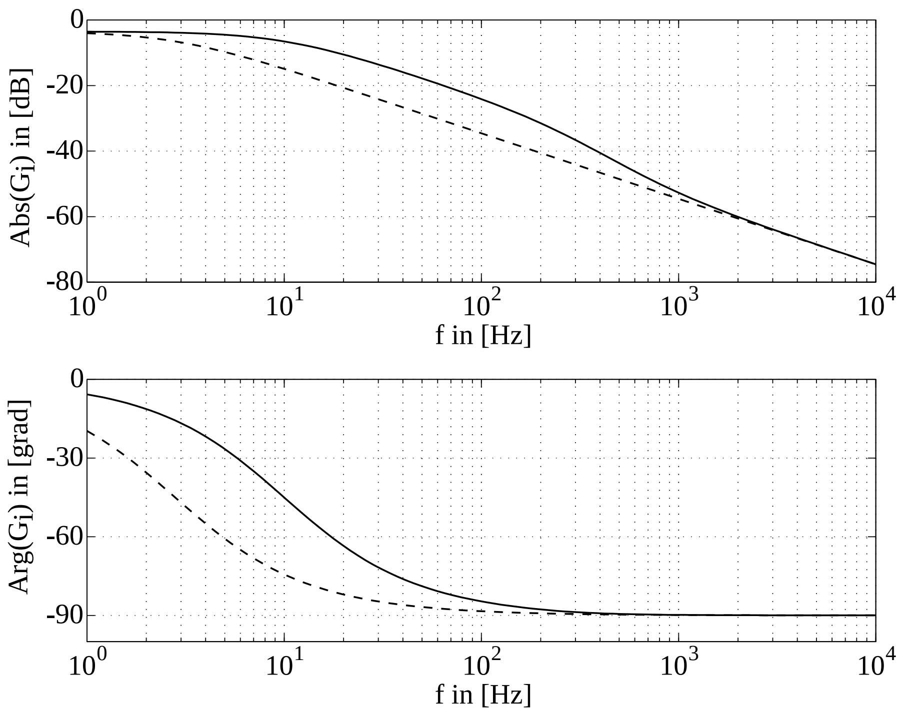

Figure 3.

The observability from the viewpoint of control theory has been proven in (Vischer, 1988), but in practice the observability is no absolute grant for stable control. A visual explanation of the observation problem is shown in figure 3. The solid line shows the open-loop transfer function G_i (equation (9)) and the dashed line shows the transfer function of the same plant but with a fixed rotor, which is equal to the transfer function G_{i0} (equation (10)). It is obvious, that the reconstruction of the rotor position can only be achieved, when these two transfer functions are different. In figure 3 it can be seen that at low and high frequencies the reconstruction of the position can't be fulfilled. The lack of position information at low frequencies is well known (Vischer, 1988) and appears as the negative stiffness of self-sensing magnetic bearings. The limiting parameters are given by the geometry of the magnetic bearings and are characterized by the time constant $\tau = (L_0+L_s)/R$ and the ratio $\xi = L_0/L_s$. In practice τ and ξ are given and cannot further be optimized. The lack of position information in high frequencies is inherent to the self-sensing magnetic bearing and hasn't been reported in literature yet. This part of the position information is not required for the principal functioning of self-sensing magnetic bearings, but it is limiting the robustness of the system.

In the following chapter a second viewpoint focused to the robustness of self-sensing magnetic bearings will be shown. As the controller tries to compensate resistance and the inductance, the identical values of G_i and G_{i0} at low and high frequencies result in a strong parameter sensitivity of the system.

PARAMETER SENSITIVITY

The first start up of a self-sensing magnetic bearing is often very difficult. The problem increases when the system parameters are measured without the rotor levitating and no additional position sensor is available for identifying the plant parameters. Considering the start up difficulties it might be surprising, that once the rotor of the self-sensing magnetic bearing is levitated, it has a relatively high robustness due to the nonlinearity of the plant. This behavior corresponds with analytical calculations and simulations. Therefore the reason for the start up difficulties is explained by the inaccurate initial values of the plant parameters on which the controller design was based. It is therefore useful to analyze the sensitivity of the controller parameters. A controller with a minimum number of parameters can be formulated as:

$$G_r(s) = \frac{u(s)}{i(s)} = \frac{b_2 s^2 + b_1 s + b_0}{s + a_0} = c_1 s + c_2 + c_3 \frac{a_0}{s + a_0} \tag{11}$$

The transfer function of the PDT$_1$-controller of equation (11) consists of an 2nd order numerator and of an 1st order denominator. In practice it is easier to realize the additive structure with the parameters c_1, c_2, c_3 and a_0 because it prevents a second derivation of the input signal. As mentioned before, the electrical parameters of the plant will be cancelled out by the controller. It is common to cancel out stable plant poles with controller zeros. Practically such a compensation consist of an residual error of about 10%. A self-sensing magnetic bearing cannot tolerate such a high residual error. An acceptable error for a self-sensing magnetic bearing to operate properly should be in the range of 1-2%, a point which will discussed later.

In the following a simple procedure for analyzing the sensitivity of the controller parameters will be presented. Due to the tied coupling between the parameters it will be here assumed that only one of the parameters is erroneous. For a given pole placement of the closed-loop system, the root locus will be calculated for every parameter of the controller. In the next step the numerical calculation searches for the admissible range of the parameter, for which the closed-loop system remains stable. This calculation will be done for all 4 parameters and for all significant pole locations of the closed-loop system. The pole locations correspond to the roots to the characteristic polynomial of equation (12) containing the variable coefficients p_1, p_2, p_3 and ω_r. Practical experience shows, that magnetic bearings should have a closed-loop dynamics with $\omega_r \approx 1.5\omega_p \ldots 3\omega_p$.

$$P(s) = s^4 + p_3\omega_r s^3 + p_2\omega_r^2 s^2 + p_1\omega_r^3 s + \omega_r^4 \tag{12}$$

The sensitivity analysis result of one specific closed-loop pole placement is shown in figure 4 (magnetic bearing parameters: ω_p = 100rad/s, L_0 = 0.35H, R and L_s are cancelled out). The solid lines show the nominal values of the controller parameters and the dashed lines represent the bounds of the controller parameter ranges for which the system remains stable. The pole placement has been optimized for a maximal distance to the bounds in addition with an high value of ω_r. Excluding ω_r from the optimization would lead to a slow system dynamics, which is not realizable in practice. An optimal controller could be found at $\omega_r \approx 0.7\omega_p$ with a pole placement including one or two very low frequency poles.

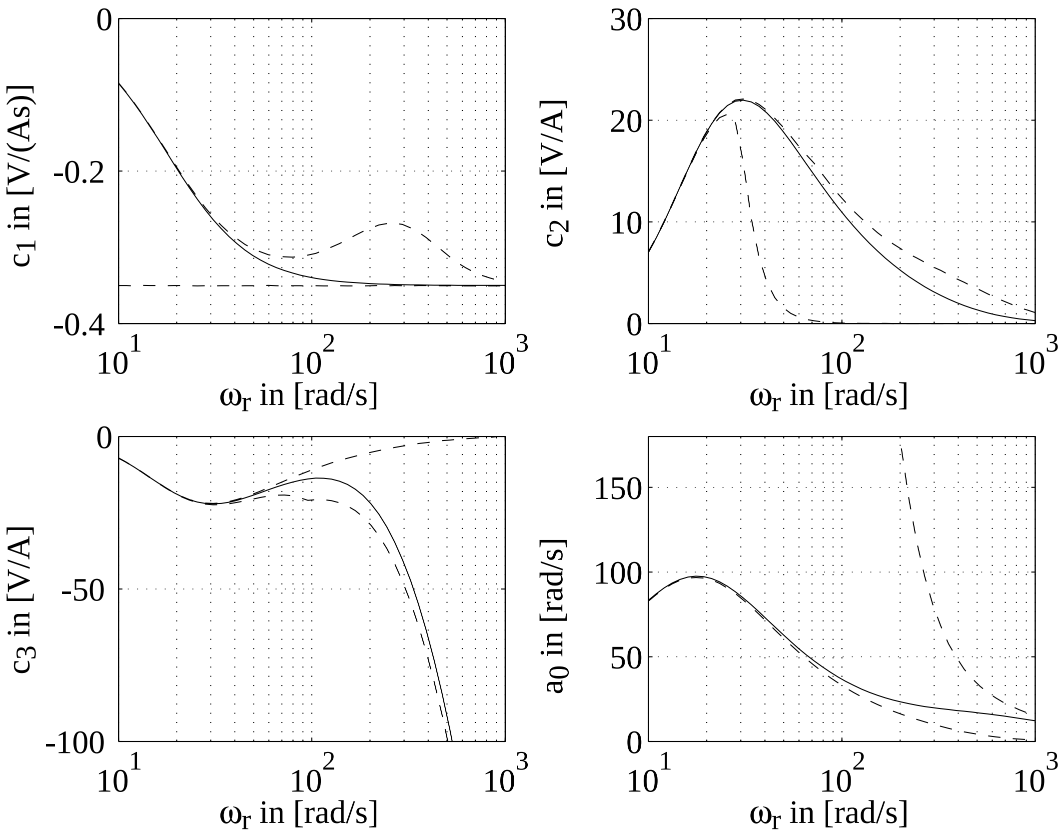

Figure 4.

As can be seen from figure 4 the range of valid controller parameter values is very small and, therefore adjustment of the controller is required. Earlier experiments showed, that it is indispensable to trim the parameter c_1. The upper bound of c_1 is corresponding with the air gap inductance L_0 and, as the nominal parameter value tends to be equal with the upper bound, this parameter is the cause for the low robustness of the self-sensing magnetic bearing. In other words a robust controller would try to completely compensate the air gap inductance L_0. In practice this can be done only with some residual error and, therefore it won't be possible to improve the controller design of the self-sensing magnetic bearing.

Trimming only c_1 won't be enough to guarantee a proper start up of the rotor levitation. It is also important to trim the sum of c_2 and c_3. But this trimming can be done without a levitated rotor because it is necessary to bring the current i to instability with the rotor fixed. Temperature sensitivity of the coil resistance R which has a similar effect like a change of c_2, needs to be compensated. This can be done by simply evaluating the bias voltage u_0, which is proportional to the coil resistance.

Once the self-sensing magnetic bearing is levitated, it is possible to further adjust the controller with a more accurate parameter identification of the plant (Gähler and Herzog, 1995).

EXPERIMENTAL RESULTS

The controller described in this paper was tested using an existing setup. The parameters of the system are $x_0 = 0.7\ 10^{-3}$m, i_0=1A, $k_s = 1.4\ 10^5$N/m, k_i = 100N/A, m = 4.6kg, L_0 = 70mH, L_s = 120mH, $R = 8\Omega$.

Due to disturbance forces the self-sensing magnetic bearing with a linear control can only react with a rotor displacement in opposite direction of the force. This negative stiffness allows a high rejection to static disturbance forces (Vischer, 1988). This property is not a special advantage of self-sensing magnetic bearings. All voltage controlled magnetic bearings can reach such a property by simply feeding back the integral of the coil currents.

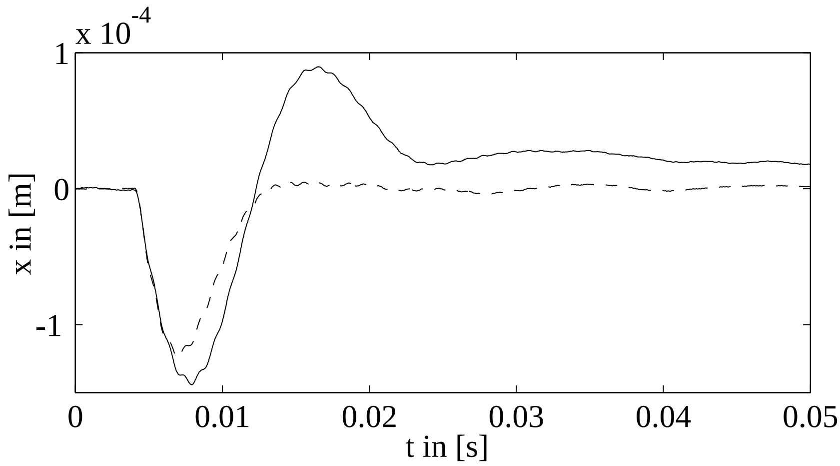

Figure 5.

It is more important to take a view on the dynamic behavior. As it can be seen from figure 5 the first part of the transient behavior is desirable. But afterwards it is followed by a slow transient time, which is caused by a low frequency closed loop pole located near the origin. The transient time could be shortened, but due to the increased parameter sensitivity this can only be done by reducing the valid range of the air gap while the rotor is levitating stable. As the analysis of the parameter sensitivity has shown, the parameter tolerance increases with one low frequency pole. Additionally the parameter tolerance can be increased with a low frequency conjugate complex pole pair near the imaginary axis. Such a pole placement has the disadvantage of tendency to oscillate at low frequencies.

SUMMARY

The self-sensing magnetic bearing with linear control extracts the information to the actual position of the rotor from the coil currents. With an appropriate control of the voltage over the coils, the rotor can be levitated. Due to the minimal amount of hardware this method leads to a relatively low cost solution, specially when the controller is built up with analogue circuit technology. The strong parameter sensitivity of this method appears with a difficult start up procedure and with a low system robustness.

In this paper a controller structure (PDT_1-controller) has been chosen. The simple structure of the PDT_1-controller allowed a deduction of an adjustment strategy helping to start up the system successfully. However, because of the strong parameter sensitivity the realization of the controller with low system performance but with stability over the full air gap is practically forced. The lack of robustness results from the slow dynamics of the closed-loop system, which is a factor 5 or more slower than the dynamics of a magnetic bearing system with additional position sensors. An increase of system performance results in a reduction of the permissible working range.

If a specific application requires higher robustness combined with the technical advantages of self-sensing magnetic bearings it is necessary to change the operational method. The self-sensing magnetic bearing with modulation method renders a real reconstruction of the rotor position and therefore a conventional method of the controller design for magnetic bearings can be used. Such a solution finally leads to a much better system performance, but also to higher costs.

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