Modal Analysis and Vibration Behaviour of a Magnetically Suspended Flywheel Energy Storage Device

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ABSTRACT

This paper presents the vibration behaviour of a flywheel energy storage device which is designed for levelling peaks in the power consumption of seam-welding machines. The system has high power (250 kW) and 1 kWh of usable energy and it is suspended by active magnetic bearings. A magnetic bearing system is unstable in nature and therefore a controller is required. For the controller design a rotordynamic model is necessary. In order to verify the Finite Element modelling of the rotor a modal analysis is done. It can be seen that, for the flywheel system presented in this paper, the axial and radial vibration modes are coupled. Further the vibration behaviour of the complete system is analyzed. It can be seen that the housing and the fixation of the stator can cause vibration problems. In order to find the origin of these vibrations an additional modal analysis of the bottom part of the housing is done. As a consequence, the design of the bottom plate is modified in order to increase system performance.

1. INTRODUCTION

The latest developments in the field of composite materials and active magnetic bearings (AMB) have led to kinetic energy storage devices which are smaller and lighter and which operate at higher rotational speed. This increase of efficiency makes kinetic energy storage devices interesting for several fields of application.

Compared with kinetic energy storage devices static energy storage devices such as batteries or capacitors have limited lifetime cycles and low power respectively low capacity.

In applications with extreme peaks in power consumption, such as seam-welding machines or island-type electrical networks, a short time energy storage device can level these peaks and, in addition, recover energy from the application.

For seam-welding machines there is a demand for an energy storage device with a capacity of 1 kWh of usable energy and high power (250 kW) of the motor/generator. This leads to a short time for loading/unloading of approximately 15 seconds.

For this reason a research project 'Kinetic Energy Storage (KIS)' was started at the ETH[1]

[1] The project partners of the ICMB are the Institute of Electrical Machines and the Chair of Power Electronics and Electrometrology. The project was funded by NEFF (Nationaler Energie Forschungs-Fond).

in 1992. The goal was to develop a kinetic short time energy storage system for stationary applications [ATvBS94].

Magnetic bearings are ideally suited for high speed and vacuum applications due to their contact free operation, low friction losses, adjustable damping and stiffness characteristics and due to the fact that no lubricants are necessary [SBT94], [SSH94]. A magnetic bearing system is unstable in nature and therefore a controller is required. For a controller design an adequate mathematical model of the plant is necessary.

For most rotordynamic problems Finite Element (FE) modelling is used. A model reduction is necessary because higher elastic modes are neither correct nor of interest and contoller design is more difficult. A modal analysis can measure eigenfrequencies and modal damping and therefore verify the mass, stiffness and damping matrices of the model. Furthermore, 'forgotten' modes can be found and modelling can be improved (see e.g. [Vig92]).

2. DESCRIPTION OF THE FLYWHEEL ENERGY STORAGE SYSTEM

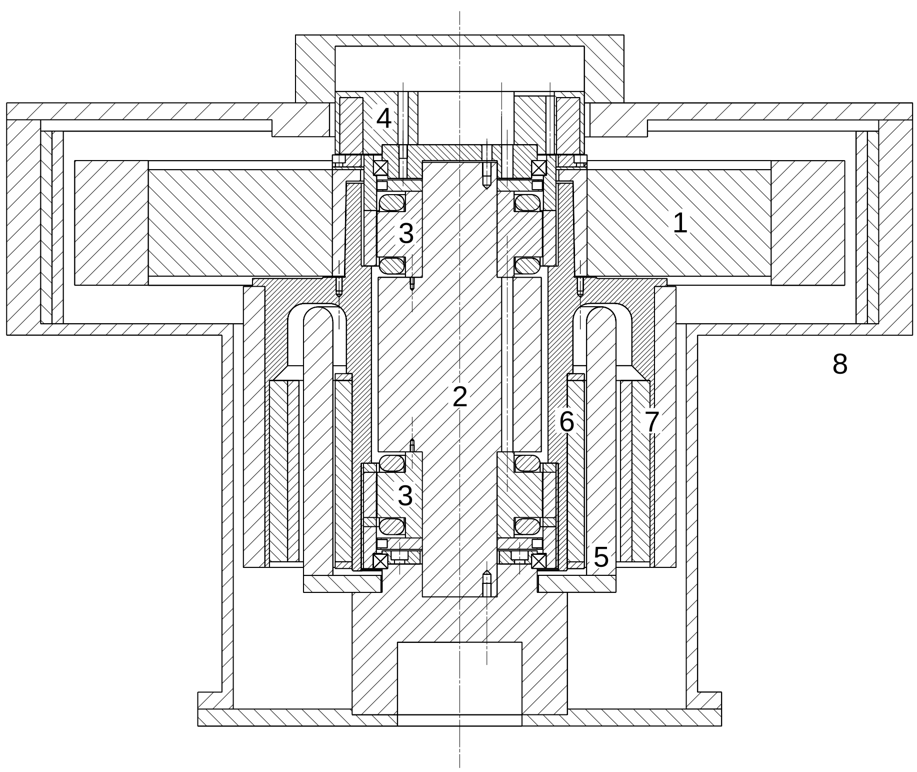

Figure 1: Cross section of the flywheel energy storage system
1: flywheel, 2: stator of the radial magnetic bearings with water cooling, 3: radial magnetic bearings, 4: thrust magnetic bearing, 5: stator of the electrical machine with water cooling, 6: inner rotor part, 7: outer rotor part, 8: vacuum housing

The system features an inner type stator with the radial magnetic bearings (see figure 1 and 2). The composite flywheel is connected to the outer type rotor with a cone interference fit. The rotor itself consists again of an inner and an outer rotor part, with the stator of the electrical machine in between. One single thrust bearing on the top of the system carries the weight of the rotor.

Table 1: Data of the KIS system

diameter of the housing	: 870 mm
height of the housing	: 810 mm
diameter of the flywheel	: 680 mm
length of the rotor	: 365 mm
total mass of the rotor	: 175 kg
nominal speed	: 15000 rpm
no-load speed	: 7500 rpm
usable energy	: 1 kWh
nominal power	: 250 kW
diameter of the radial AMBs	: 148 mm
nominal air gap	: 0.6 mm
air gap of the retainer bearings	: 0.3 mm
maximum (static) thrust AMB force	: 2000 N
maximum (static) radial AMB force	: 400 N
sampling frequency of the AMB controller	: 4 kHz

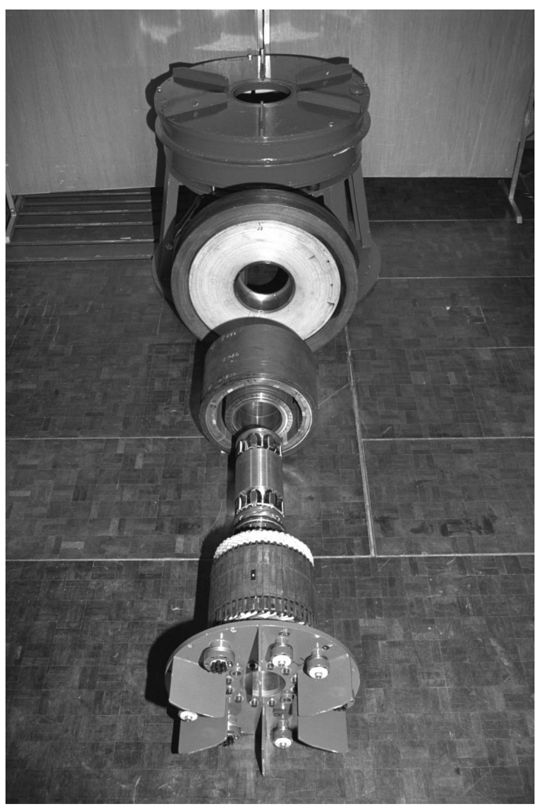

Figure 2: The kinetic energy storage system KIS
From above: housing, flywheel, rotor, stator of the radial magnetic bearings, stator of the electrical machine

3. MAGNETIC BEARINGS

The basic principle of active electromagnetic bearings is as follows (see figure 3). A position sensor measures the rotor displacement from a nominal position. This signal is given to a controller which determines the appropriate input of the power amplifiers. The generated currents generate the magnetic bearing forces acting on the rotor. For the KIS system the controller is realized on a digital signal processor (DSP).

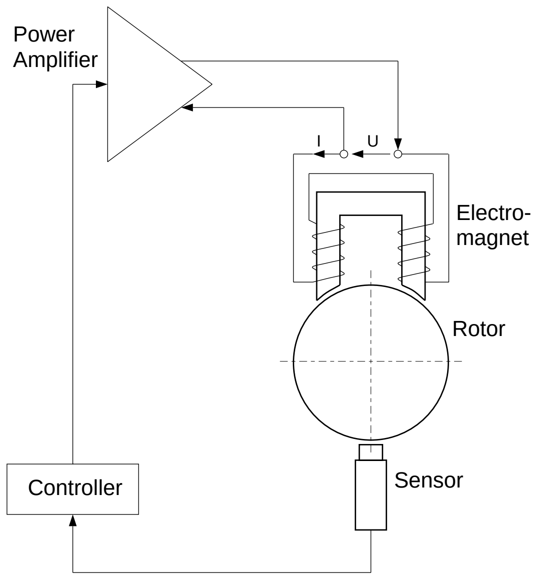

Figure 3: The basic principle of active magnetic bearings

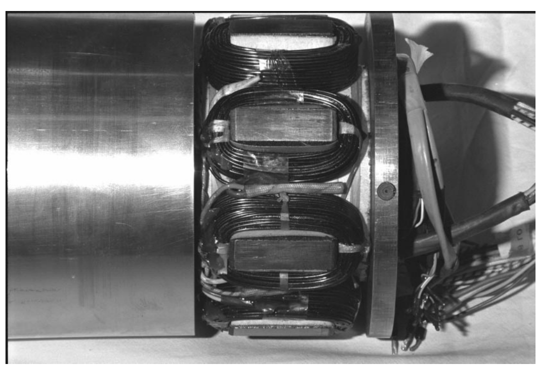

Figure 4: A radial bearing and a carrier ring with the radial sensors

Position signals, amplifier currents and controller parameters are available on the DSP and can be analyzed online or stored and analyzed later. Hence, magnetic bearings can be used for monitoring and for measuring the transfer functions of the plant. In this case the bearing is used as actuator generating a harmonic force $F \sin(\omega t)$ and the response is measured with the displacement sensors. These measured transfer functions include the bearing behaviour (actuator, sensor and amplifier characteristics) and the complete mechanical part (rotor and stator/housing). These measurements yields important informations on the vibration behaviour of the system but cannot replace modal analysis. In radial direction the magnetic bearing system has only two inputs and two

outputs which is not sufficient to identify the mode shapes. Furthermore, housing and rotor modes cannot be distinguished.
It is clear that measurements with magnetic bearings can only be done when the rotor levitates which requires a model based controller. Therefore, good modelling of the rotor and the bearings is necessary as well as a verification of the model with modal analysis.

4. ROTORDYNAMIC MODEL

In order to achieve a realistic model Finite Element analysis is used. The program MADYN [Ing90] is especially made for rotordynamic applications and uses Timoshenko-beams as elements. The number of nodes depends on the geometry (see figure 5) of the rotor and the number of eigenmodes to be reproduced properly. A critical point is the modelling of shrink-fitted parts e.g. lamination sheets of the bearing or the motor. These parts are modelled as additional masses, where in a first step an increase of the rotor stiffness is neglected. The same is done for the flywheel. The rotor is modelled with two rotors, represented by the rotor axes, which are connected stiff. Due to the complexity of the system a modal analysis is advisable in order to verify the model.

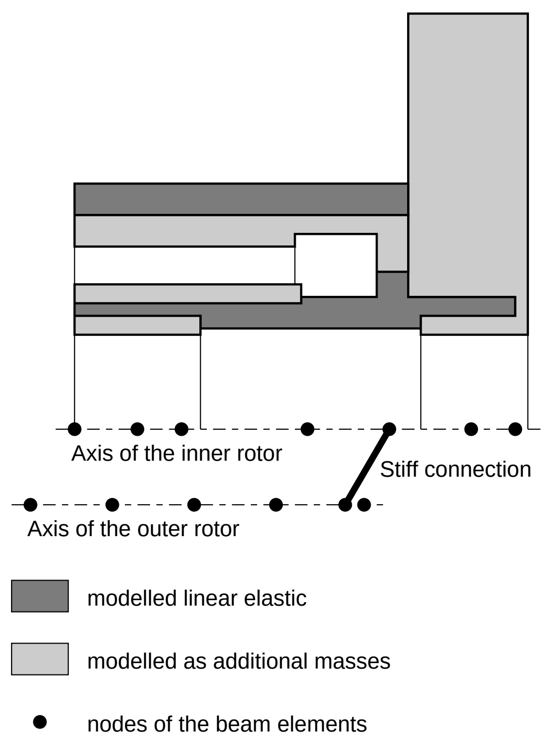

Figure 5: Finite Element model of the rotor

The rotor can be described as a $\boldsymbol{MDGK}$-system (see equation 1). The gyroscopic matrix $\boldsymbol{G}$ describes the coupling between the rotor axes while rotating. The KIS rotor has a strong gyroscopic coupling and therefore the gyroscopic matrix $\boldsymbol{G}$ has to be considered for the controller design [AK95].

The equations of motion can be written as:

$$\boldsymbol{M}\ddot{\boldsymbol{x}} + (\boldsymbol{D} + \Omega\boldsymbol{G})\dot{\boldsymbol{x}} + \boldsymbol{K}\boldsymbol{x} = \boldsymbol{F} \qquad (1)$$

In equation 1 $\boldsymbol{M}$ is the symmetric mass matrix, $\boldsymbol{D}$ the symmetric inner damping matrix, $\boldsymbol{G}$ the skew-symmetric gyroscopic matrix, $\boldsymbol{K}$ the symmetric stiffness matrix, Ω the rotational speed of the rotor and $\boldsymbol{F}$ the vector of the external forces including the magnetic bearing forces.

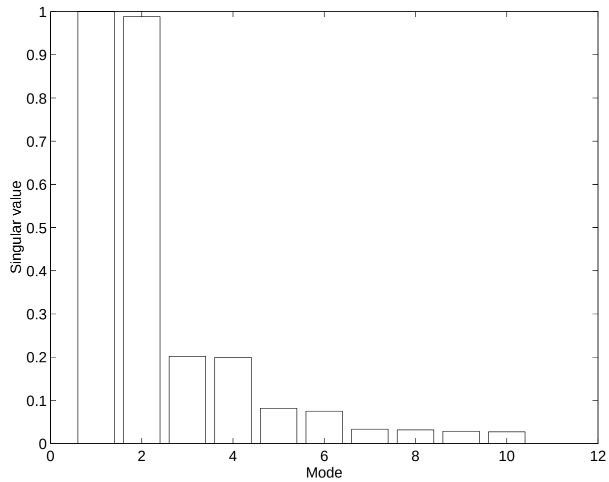

Figure 6: Singular values of the controllability gramians of the first ten modes of the FE model based system (normalized to the first mode)

When the controllability gramians $G_c = \int_0^\infty e^{\boldsymbol{A}\tau}\boldsymbol{B}\boldsymbol{B}'e^{\boldsymbol{A}'\tau}\mathrm{d}\tau$ are calculated for the FE model it can be seen that a modal reduction to two or four modes is useful (see figure 6). This leads to a rigid body model or a model which includes the first and second bending mode.
The modal reduction is briefly shown for one rotor plane. More information can be found in [Ewi92], [Lar90]. A transformation from geometrical/physical coordinates $\boldsymbol{x}$ to modal coordinates $\tilde{\boldsymbol{x}}$ is made. $\boldsymbol{\Phi}$ is the eigenvector matrix in ascending order.

$$\boldsymbol{x} = \boldsymbol{\Phi}\tilde{\boldsymbol{x}} \tag{2}$$

$$\tilde{\boldsymbol{M}} = \boldsymbol{\Phi}^T\boldsymbol{M}\boldsymbol{\Phi} = \boldsymbol{I} \tag{3}$$

$$\tilde{\boldsymbol{K}} = \boldsymbol{\Phi}^T\boldsymbol{K}\boldsymbol{\Phi} = \mathrm{diag}(\omega_j^2 + \delta_j^2) \tag{4}$$

$\boldsymbol{I}$: identity matrix, ω_j: eigenfrequency of the j-th mode, δ: damping factor
$\tilde{\boldsymbol{D}}$ is diagonal, when modal (structural) damping is assumed.

$$\tilde{\boldsymbol{D}} = \boldsymbol{\Phi}^T\boldsymbol{D}\boldsymbol{\Phi} = 2\mathrm{diag}(\delta_j) \tag{5}$$

$$\tilde{\boldsymbol{G}} = \boldsymbol{\Phi}^T\boldsymbol{G}\boldsymbol{\Phi} \tag{6}$$

$$\tilde{\boldsymbol{B}}_z = \boldsymbol{\Phi}^T\boldsymbol{B}_z \tag{7}$$

$$\tilde{\boldsymbol{C}} = \boldsymbol{C}\boldsymbol{\Phi} \tag{8}$$

Now a reduced eigenvector matrix $\boldsymbol{\Phi}_r$ including k modes is used.

$$\boldsymbol{\Phi}_r = [\boldsymbol{\Phi}_1, \ldots, \boldsymbol{\Phi}_k] \tag{9}$$

The reduced order plant model for both rotor planes can be written as in equations 10 and 11. The index 1 or 2 represents the x-z respectively the y-z plane.

$$\begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I} \end{bmatrix} \begin{bmatrix} \ddot{\tilde{\boldsymbol{x}}}_{r,1} \\ \ddot{\tilde{\boldsymbol{x}}}_{r,2} \end{bmatrix} + \left[\begin{bmatrix} \tilde{\boldsymbol{D}}_r & \boldsymbol{0} \\ \boldsymbol{0} & \tilde{\boldsymbol{D}}_r \end{bmatrix} + \Omega \begin{bmatrix} \boldsymbol{0} & \tilde{\boldsymbol{G}}_r \\ -\tilde{\boldsymbol{G}}_r & \boldsymbol{0} \end{bmatrix} \right] \begin{bmatrix} \dot{\tilde{\boldsymbol{x}}}_{r,1} \\ \dot{\tilde{\boldsymbol{x}}}_{r,2} \end{bmatrix} + \begin{bmatrix} \tilde{\boldsymbol{K}}_r & \boldsymbol{0} \\ \boldsymbol{0} & \tilde{\boldsymbol{K}}_r \end{bmatrix} \begin{bmatrix} \tilde{\boldsymbol{x}}_{r,1} \\ \tilde{\boldsymbol{x}}_{r,2} \end{bmatrix} = \begin{bmatrix} \tilde{\boldsymbol{B}}_{z,r} & \boldsymbol{0} \\ \boldsymbol{0} & \tilde{\boldsymbol{B}}_{z,r} \end{bmatrix} \begin{bmatrix} \boldsymbol{f}_{r,1} \\ \boldsymbol{f}_{r,2} \end{bmatrix} \tag{10}$$

$$\begin{bmatrix} \boldsymbol{y}_1 \\ \boldsymbol{y}_2 \end{bmatrix} = \begin{bmatrix} \tilde{\boldsymbol{C}}_r & \boldsymbol{0} \\ \boldsymbol{0} & \tilde{\boldsymbol{C}}_r \end{bmatrix} \begin{bmatrix} \tilde{\boldsymbol{x}}_{r,1} \\ \tilde{\boldsymbol{x}}_{r,2} \end{bmatrix} \tag{11}$$

This reduced system can be used for controller design. With a modal analysis of the rotor eigenfrequencies and modal damping can be measured and, therefore, $\tilde{\boldsymbol{K}}_r$ and $\tilde{\boldsymbol{D}}_r$ can be verified.

5. MODAL ANALYSIS OF THE ROTOR

One-dimensional geometry models representing the rotor axis are sufficient for symmetrical rotors to identify all bending modes. Due to the more complex geometry of the KIS rotor a two-dimensional geometry model representing the cross section of the rotor is used. Measurements show that the vibration behaviour of the flywheel in axial direction has an enormous influence on the vibration behaviour of the rotor in radial direction. Therefore, an additional modal analysis of the flywheel is made. All measurements are done with an impact hammer excitation using a Difa Scadas signal analyzer and SDRC I-DEAS software [SDR94]. A two-dimensional model is used in order to identify the membrane modes of the flywheel. An additional FE analysis of the vibration behaviour of a thin shell with fixed inner edge is made with the general purpose FE program MARC [MAR92]. This allows a comparison of the flywheel mode shapes with the calculated mode shapes for an ideal membrane. The FE analysis leads to identical modes shapes but to different eigenfrequencies. The main problem here is to define proper boundary conditions and material properties of the composite flywheel. The eigenfrequencies of the membrane modes depend on the fixation and therefore on the rotor. Table 2 shows the eigenfrequencies of the KIS rotor. A second modal analysis is done with the modified rotor. Here, the permanent magnets (PM) of the electrical machine with a total weight of 36 kg are removed from the rotor. It can be seen that all eigenfrequencies of the rotor including the membrane modes of the flywheel depend on the rotor mass.
The first two and totally four of the first six measured radial eigenfrequencies are forced by axial modes of the flywheel (see figure 7). The first bending mode of the rotor is only the third measured eigenfrequency. This measured mode shape is identical to the mode shape calculated with MADYN. But the eigenfrequency calculated with MADYN is slightly too high. A variation of the FE model shows that the stiffness of the outer rotor part (which

consists of steel and carbon fibre) has been assumed too high. The sixth eigenfrequency is not a bending mode because the left and the right part of the inner rotor have opposite phase. This mode shape is like the vibration of a bell. Neither this mode nor the coupling of axial and radial vibration modes (membrane modes of the flywheel) can be modelled with MADYN. On the other hand general purpose FE programs are not able to calculate the gyroscopic matrix which is crucial for flywheel systems. This modelling problem is hard to overcome, one possibility is to identify the gyroscopic matrix (see [HZ92], [WK94], [Nor84], [Mah84], [FRGN95]).

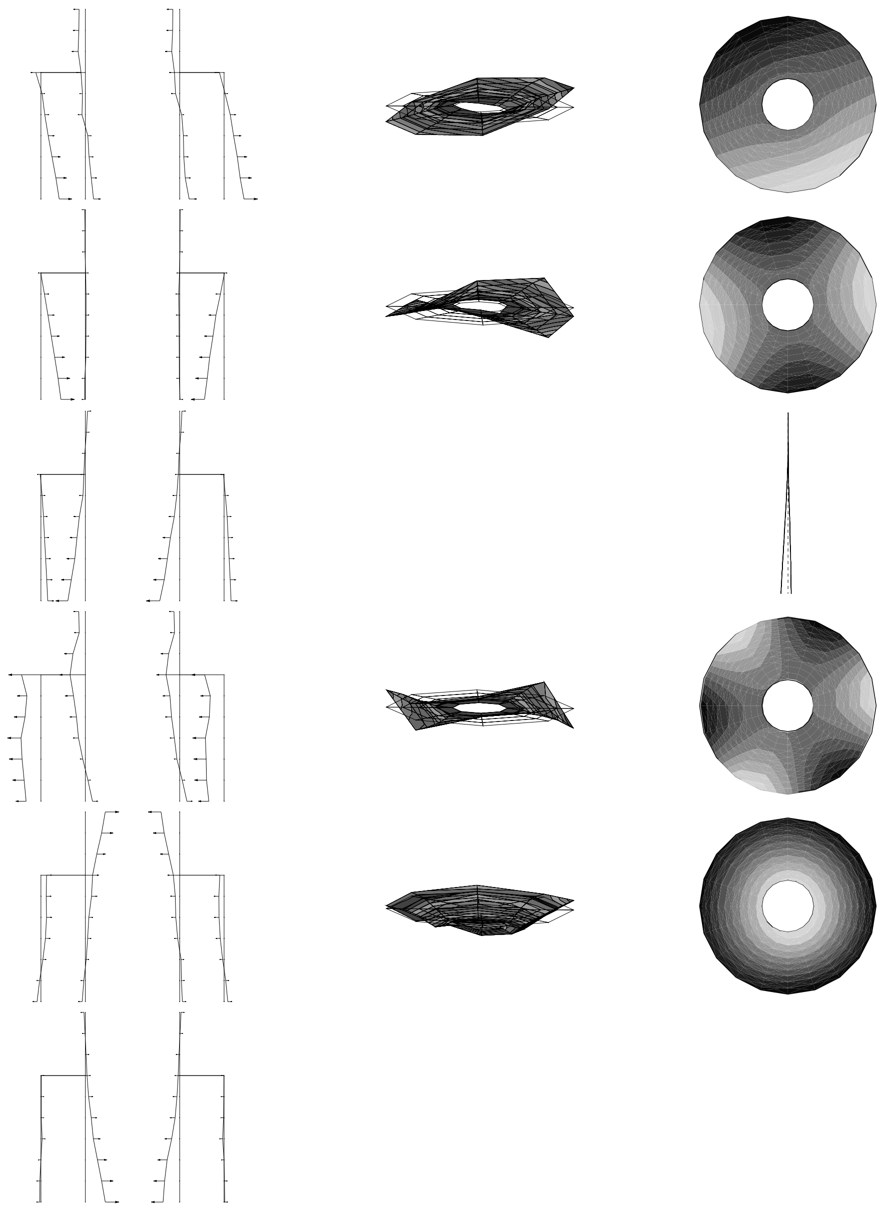

Figure 7: Mode shapes of the first six rotor eigenfrequencies
left column: radial mode shapes, middle column: axial membrane mode shapes, right column: axial and radial mode shapes calculated with FE analysis

Table 2: Eigenfrequencies (EF) and modal damping of the KIS rotor

Mode	EF of the KIS rotor	Modal damping	EF without PM
1. mode	484 Hz	0.47 %	581 Hz
2. mode	631 Hz	0.38 %	819 Hz
3. mode	909 Hz	0.53 %	1109 Hz
4. mode	1672 Hz	0.72 %	1831 Hz
5. mode	1905 Hz	0.23 %	2095 Hz
6. mode	2333 Hz	0.76 %	2476 Hz

For the KIS project a rigid body model is used for the controller design. This is possible because the first eigenfrequency is 484 Hz and therefore nearly two times higher than the maximum rotational speed. Although a rigid body model is used for controller design the knowlegde of the rotor vibration modes and the radial eigenfrequencies is important for further vibration analysis of the complete system.

6. VIBRATION BEHAVIOUR OF THE COMPLETE SYSTEM

The coupling between rotor and housing due to the magnetic bearings is weak and therefore the rotor eigenfrequencies do not change significantly. Therefore, eigenfrequencies of the complete system (i.e. rotor and stator/housing) which are not rotor eigenfrequencies have to be caused by the housing respectively the stator. For most applications the housing can be assumed as rigid and the influence on the vibration behaviour of the complete system is negligible. In the following it will be shown that the housing of the KIS system has an enormous influence on the behaviour of the system.

Due to the fact that magnetic bearings are active elements vibrations can be damped or excited. The position sensor measures the displacement between stator and rotor. Therefore, not only rotor modes but also housing modes may have big influence on the vibration behaviour of the system. Measurements of the KIS system show that there are mainly two housing eigenfrequencies which have big influence on the system behaviour. In figure 8 a resonance can be seen in the lower radial bearing transfer function at approximately 220 Hz. In the upper radial bearing a resonance of approximately 400 Hz can be seen.

The 400 Hz mode depends on the fixation of the stator of the radial bearings. When the fixation is less stiff (e.g. due to detaching some screws) this eigenfrequency can decrease drastically. Simulations show that the first bending eigenfrequency of the stator is 977 Hz with two fixed edges and 125 Hz with one fixed egde. The fixation of the stator of the radial bearings is therefore important and has to be considered in the design. For the KIS project the 400 Hz mode does not cause serious problems.

The crucial problem has been the vibration mode at 220 Hz. Even small excitation amplitudes at this frequency have led to very large vibration amplitudes and rotor drop downs. Therefore, a sharp low-pass filter has been necessary in order to keep the controller gain in this frequency range small. But even with this filter the system behaviour has been poor. When the excitation has exceeded a certain value the vibrations did not stop even when the excitation was stopped. Due to the filter the magnetic bearings have introduced negative damping to the system in this frequency range.

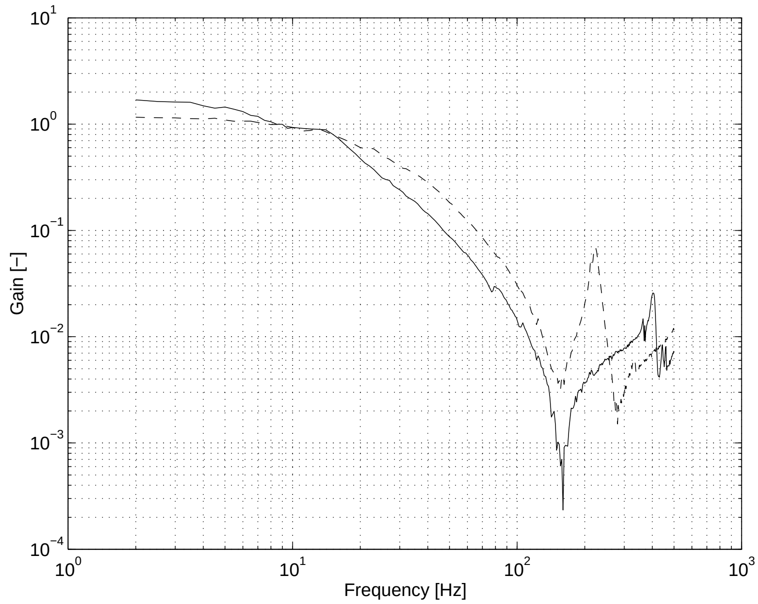

Figure 8: Measured transfer function of the plant
solid line: upper radial bearing
dashed line: lower radial bearing

Normally unbalance is the most important disturbance force for rotor systems. Due to the high power of the electrical machine other harmonic disturbance forces occur. These excitations may lead to the vibration problem described above. It is clear that this behaviour would be an unacceptable risk at high rotational speed. Moreover, the strong gyroscopic coupling of the KIS rotor requires a controller which depends on the rotational speed [AK95]. Such a controller cannot be realized with the filter which is necessary for this vibration mode. Hence, even with reduced excitation forces (e.g. improved balancing of the rotor or unbalance compensation [LH94]) the maximum rotational speed cannot be reached. Therefore, this vibration problem has led to an extreme limitation of the rotational speed.

7. MODAL ANALYSIS OF THE BOTTOM PART

The origin of this mode has been found at the bottom part of the housing. Therefore, a modal analysis of this part is made. The result shows that the bottom plate of the housing has a flexural mode at 220 Hz. The stator of the electrical machine is connected stiff with the bottom plate and has a rigid body motion. Due to the fact that the stator of the radial magnetic bearings is fixed at the bottom plate the vibration mode has a strong influence on the magnetic bearings. The stator of the electrical machine itself has an eigenmode at 420 Hz which has only little influence on the bottom plate and, therefore, on the magnetic bearing behaviour.

As a result of the modal analysis the design of the bottom plate has been improved. The thickness of the plate has been increased and screw-holes have been reduced if possible. Due to the increased stiffness of the bottom plate the frequency of the crucial mode has increased from 220 Hz to 370 Hz. The resonance amplitude has decreased and, therefore, larger excitation amplitudes are possible. With the improved vibration behaviour it is possible to use the desired controller which is necessary to reach the maximum rotational speed. The increase of system performance can be clearly seen at the measured transfer functions (see figure 10 and figure 8).

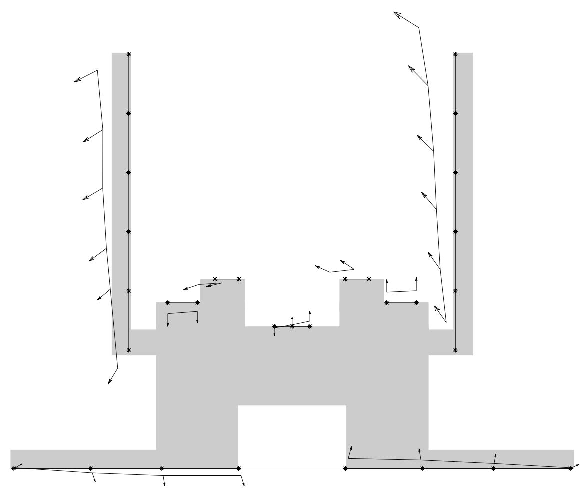

Figure 9: Mode shape of the crucial vibration mode of the bottom plate

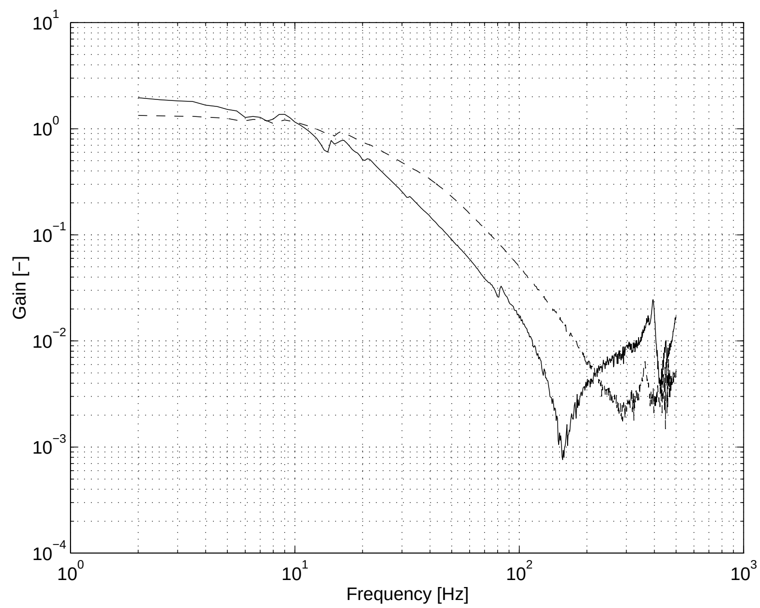

Figure 10: Measured transfer function of the plant
solid line: upper radial bearing
dashed line: lower radial bearing

8. CONCLUSIONS

The axial vibration modes of a flywheel may strongly influence the radial vibration behaviour of the rotor. This has to be considered in the rotor design. Modal analysis can identify these modes and provide important information on the vibration behaviour of the rotor. For flywheel systems these coupled modes may cause serious modelling problems.

For magnetic bearing systems the vibration behaviour of the housing may have enormous influence on the system performance. Housing vibrations of the KIS system led to a restricted controller design and limited the maximum rotational speed. The origin of these vibrations could be found with modal analysis of the bottom part. With an improved design of the bottom plate the system performance was increased and the rotational speed was no longer limited.

REFERENCES

[AK95] M. Ahrens and L. Kučera. Cross Feedback Control of a Magnetic Bearing System. In *3rd International Symposium on Magnetic Suspension Technology*, Tallahassee, 1995.

[ATvBS94] M. Ahrens, A. Traxler, P. von Burg, and G. Schweitzer. Design of a Magnetically Suspended Flywheel Energy Storage Device. In *4th International Symposium on Magnetic Bearings*, Zurich, 1994.

[Ewi92] D.J. Ewins. *Modal Testing: Theory and Practice.* Research Studies Press, Taunton, 1992.

[FRGN95] P. Förch, A. Reister, C. Gähler, and R. Nordmann. Modale Analyse an rotierenden Maschinen mittels Magnetlager. In *Schwingungen in Rotierenden Maschinen*, Kaiserslautern, 1995.

[HZ92] R.P.S. Han and J.W.Z. Zu. Modal Analysis of Rotating Shafts. *Journal of Sound and Vibration*, 156, 1992.

[Ing90] Ingenieurbüro Klement. *Eingabebeschreibung MADYN*, 1990.

[Lar90] R. Larsonneur. *Design and Control of Active Magnetic Bearing Systems for High Speed Rotation.* PhD thesis, ETH Swiss Federal Institute of Technology, Zurich, 1990.

[LH94] R. Larsonneur and R. Herzog. Feedforward Compensation of Unbalance: New Results and Application Experiences. In *IUTAM Symposium The Active Control of Vibration*, Bath, 1994.

[Mah84] O. Mahrenholtz. *Dynamics of Rotors, Stability and System Identification.* Springer-Verlag, Wien, 1984.

[MAR92] MARC Analysis Research Corporation. *User Manuals MARC*, 1992.

[Nor84] R. Nordmann. Modal Analysis in Rotor Dynamics. In *CISM Dynamics of Rotors*, Udine, 1984.

[SBT94] G. Schweitzer, H. Bleuler, and A. Traxler. *Active Magnetic Bearings.* vdf, Zurich, 1994.

[SDR94] SDRC Structural Dynamics Research Corporation. *User Manuals I-DEAS*, 1994.

[SSH94] G. Schweitzer, R. Siegwart, and R. Herzog, editors. *Fourth International Symposium on Magnetic Bearings*, Zurich, 1994.

[Vig92] F. Viggiano. *Aktive magnetische Lagerung und Rotorkonstruktion elektrischer Hochgeschwindigkeits-Antriebe.* PhD thesis, ETH Eidgenössische Technische Hochschule, Zürich, 1992.

[WK94] W. Wang and J. Kirkhope. New Eigensolutions and Modal Analysis for Gyroscopic/Rotor Systems. *Journal of Sound and Vibration*, 175, 1994.